

Inches 1 2 3 4 5 6 7 8

Centimetres 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

KODAK Color Control Patches © The Tiffen Company, 2000
 LICENSED PRODUCT
 3/Color Black

Blue	Cyan	Green	Yellow	Red	Magenta	White
1	2	3	4	5	6	7
8	9	10	11	12	13	14

A 1 2 3 4 5 6 **M** 8 9 10 11 12 13 14 15 **B** 17 18 19



TAJIMA JAPAN

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

門 二奴 2
號 691
卷 一 2

官許

金澤關口開著述

初 篇
點竄問題集

完

明治五壬申三月出版

卧龍房藏



此卷者米利堅^{リヂヤ}の^イ氏英吉利^イの^トドホント^ル氏
等ノ諸書ヲ取テ以テ編輯ス然ルニ書中洋文字
ヲ用テ題スト雖^レ書体洋書ト逆^ルルハ下卷譯
文ヲ以テ題スル故也諸賢其順逆不正ナルヲ答
ムル^レ無^クハ幸甚

明治五壬申三月一日
河村^{カムラ}氏寄贈

- (1) $ab + cb.$
- (2) $ad + b - d.$
- (3) $bc + ab - c.$
- (4) $(bc + a)b.$
- (5) $(bd - a)(ac - d).$
- (6) $(d + c)(d - c).$
- (7) $\frac{a + 2b}{c} + d.$
- (8) $(a^2 - b)(c + d).$
- (9) $abc + cd + ad.$
- (10) $\frac{a + bc}{d} \times (c + d).$

Find the numerical values of the following expressions, when, $a = 1, b = 2, c = 3, d = 4$

- (11) $ad - c + b.$
- (12) $ab + bc - d.$

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記号用法

when, $a=5, b=2, c=4, d=3.$

$$(26) \quad \frac{6}{a} - \frac{3}{b} + \frac{10}{c-d} - \frac{14}{c+d}.$$

$$(27) \quad \left(\frac{a^2b}{c} \times d\right) \div \left(\frac{ab^2}{c} + d\right).$$

$$(28) \quad \frac{a^2+b^2-d^2}{a+b+d} + \frac{abcd}{2b+c} - \frac{4a^2-7bc+2}{2c+d}.$$

$$(29) \quad \frac{12(a+b^2)}{d^3} - (c-b) + \left(\frac{7}{a^2-b^2} \times \frac{74}{c^2-d^2}\right).$$

$$(30) \quad \{[a+b \times c+d] \div b+a\} \times c.$$

$$(73) \quad (a+b)c^2-d.$$

$$(74) \quad (a+b)(d-b).$$

$$(75) \quad (ab+ad)c+d.$$

$$(76) \quad (ab+c)(ad-a).$$

$$(77) \quad 3a^2b^2-2(a+d+1).$$

$$(78) \quad \frac{a+c}{2} \times (a+d).$$

$$(79) \quad \frac{a^2+b^2+c^2}{7} \times \frac{a^3+b^3+c^3-d}{2}.$$

$$(20) \quad \frac{ab^4-c-a^3}{6} \times \frac{4a^2-b+d^3}{33}.$$

when, $a=4, b=3, c=2, d=7.$

$$(21) \quad 5\left(\frac{ab}{3} - \frac{a-d}{3}\right).$$

$$(22) \quad |(a^2b+1)d| \div (a^2b+d).$$

$$(23) \quad 4\left(abc - \frac{b^3}{9}\right) \times (30c^3-ab^3d^3).$$

$$(24) \quad \frac{a+b+c}{a-b+d} + \frac{abcd}{ab} \div \frac{4a^2+b^2-d^2}{bc+b}.$$

$$(25) \quad \frac{75(a+a+b)}{3c^2} - \frac{a-c}{2} + \frac{3}{abd} \times a^3b^3c^3d^3.$$

Addition.

(3)	(2)	(7)
+ 4a ² bc	- 2a ² bc	+ 7a ² bc
- 2a ² bc	- 3a ² bc	+ a ² bc
+ 7a ² bc	- a ² bc	+ 3a ² bc
<u>- 5a²bc</u>	<u>- 8a²bc</u>	<u>+ 5a²bc</u>

(6)	(5)	(4)
c + bx ² + d	3a - 3bx	- 8a ² bc
4c - 2bx ² - 2d	9a - 5bx	+ 5a ² bc
<u>5c + 3bx²</u>	<u>5a - 4bx</u>	- 2a ² bc
		<u>+ 3a²bc</u>

(8)	(7)
4a + bc + 5d	3x ² y - 3y ² x - 4y + 2
2a + 2bc + 3d	3x ² y + 7y ² x - 8y
<u>3a - 3bc</u>	<u>8x²y - 5y²x + 5y</u>

(75) add $x^3 - y^3 + 2xy^2 - 3x^2y$, $2x^3 + 2y^3$
 $- 3xy^2 - 5x^2y$, $6x^2y + 6xy^2$
 $- x^3 - y^3$, and $5xy^2 - 2y^3 - 4x^3$
 $+ 8x^2y$.

(76) " $2x + 3y - 4z - 10$, $8y - 4x$
 $+ 7z + 8$, $11x + 5y - 70y - 2$,
and $16 + 10x + 12y + 14z$.

(77) " $3x^3 + 2y^3 + z^3 + 8xyz$, $2x^3$
 $+ y^3 - 3z^3 - 4xyz$, $z^3 + 3x^3$
 $- 2y^3 - 2xyz$, and $xyz + x^3$
 $+ y^3 + z^3$.

(78) " $x^4 + 3x^3y + x^2z - 2xv$, $30x^4$
 $- 29x^2z + 78xv - 17x^3y$, $16xv$
 $+ 22x^3y - 15x^4 - 32x^2z$, and
 $17x^2z - 12x^4 + 6x^3y - 11xv$.

(79) " $ax - by$, $x - y$, $ax - x$, and
 $ax + x$.

(70)

$$\begin{aligned} &4ab - 4c + 2(a+b) \\ &3ab + 5c + 5(a+b) \\ &ab + c + 3(a+b) \\ &\hline -2ab + 7c - 4(a+b) \\ &\hline -ab - c - 2(a+b) \end{aligned}$$

(9)

$$\begin{aligned} &4cx^2 + 5dy^2 - 2z^3 + d \\ &3cx^2 + 2dy^2 - 2z^3 \\ &\hline -2cx^2 - 5dy^2 + 5z^3 \end{aligned}$$

(72)

$$\begin{aligned} &a + b + c \\ &a + b - c \\ &a - b + c \\ &\hline -a + b + c \end{aligned}$$

(77)

$$\begin{aligned} &12x^2y + 2(a+b)z^2 \\ &- 77x^2y - (a+b)z^2 \\ &4x^2y + 4(a+b)z^2 \\ &- 3x^2y + 2(a+b)z^2 \\ &\hline x^2y + (a+b)z^2 \end{aligned}$$

(74)

$$\begin{aligned} &2a^2 - 17ab + 3b^2 \\ &5a^2 + 12ab - 5b^2 \\ &12a^2 + 6ab - 9b^2 \\ &\hline 3a^2 + 6ab + 3b^2 \end{aligned}$$

(73)

$$\begin{aligned} &2ax + 3by \\ &3ax + 2by \\ &7ax + by \\ &\hline 8ax + 7by \end{aligned}$$

$$(26) \text{ add } -b+3c-d-175e+6f-5g, \\ 3b-2c-3d-e+27f, 5c \\ -8d+3f-7g, -7b-6c+ \\ 17d+9e-5f+77g, -3b \\ -5d-2e+6f-9g+h.$$

$$(27) \text{ '' } 7a^2b-3abc-8b^2c-9c^3 \\ +cd^2, 8abc-5a^2b+3c^3- \\ 4b^2c+cd^2, 4a^2b-8c^3-3d^3 \\ +9b^2c.$$

$$(28) \text{ '' } 5a^2bc+6bx-4af, -3a^2bc \\ -6bx+14af, -af+9bx+ \\ 2a^2bc, 6af-8bx+6a^2bc.$$

$$(29) \text{ '' } a^2n^2+3a^3m+b, -6a^2n^2-b \\ -6a^3m, 9b-9a^3m-3a^2n^2, \\ 7mn \quad 5b-2a^3m+3a^2n^2, \\ 12a^3m+5a^2n^2+2b \quad 3mn, \\ a^2n^2+mn-a^3m.$$

$$(20) \text{ add } ax+2bx+4by-3ay, 2ax \\ +bx+2ay-by, \text{ and } 4ax \\ +3by.$$

$$(21) \text{ '' } px+qy+rx-c, 2px-2qy \\ +2c, 3qy-px+4c, \text{ and } \\ 7px-8qy-rx-3c.$$

$$(22) \text{ '' } ax^2+a^2x-2ax, x-ax+ \\ 2x^2, ax^2-2x+x^2 \text{ and } -2ax \\ -2a^2x-2ax^2.$$

$$(23) \text{ '' } a^2x-ax^2-x^2, ax-x^2-a^2, \\ -2a^2-2a^2x-2ax^2, \text{ and } \\ -3a^2x+3a^2+3ax^2.$$

$$(24) \text{ '' } a-x+4y-3z+w, x-w \\ -y-3a-2x, \text{ and } x+y \\ +z.$$

$$(25) \text{ '' } ax^2y+bx^2y^2+cxz^3, dxy^2z^2 \\ +oxz^3, \text{ and } 2ax^2y+4bdx.$$

Subtraction.

(3)	(2)	(1)
$8a^2bc$	$13a^2b$	$5a^2bc$
<u>$4a^2bc$</u>	<u>$9a^2b$</u>	<u>$2a^2bc$</u>

(6)	(5)	(4)
$10b^2d$	$7ac$	$72ab$
<u>$-3b^2d$</u>	<u>$-4ac$</u>	<u>$6ab$</u>

(9)	(8)	(7)
$6a^2 - 8b$	$-3a^4b^n$	$-8a^2bc$
<u>$3a^2 - 5b$</u>	<u>$-5a^4b^n$</u>	<u>$+3a^2bc$</u>

(11)	(10)
$4xy^2 + 4z$	$3x^2 - 4x^2y + 8$
<u>$-3xy^2 + 7z - 6x^3$</u>	<u>$5x^2 - 6x^2y - 3$</u>

(30) add $4a^3bc - 76a^4x - 9ax^3d, 6a^3bc$
 $- 6ax^3d + 17a^4x, 76ax^3d - a^4x$
 $- 9a^3bc, 25a^4x + 4a^3bc +$
 $4ax^3d.$

$$(23) 2x^3 - 3xy^2 + 2y^2x - (x^3 + y^3 - y^2x).$$

$$(24) 3x - \{x - 3a - (2y - a)\}.$$

$$(25) a^2 - (b^2 - c^2) - \{b^2 - (c^2 - a^2)\} + c^2$$

$(b^2 \quad a^2)$

$$(26) x + y + z - (x - y) - (y - z) - (-y).$$

$$(27) 2(a - b) - c + d - \{a - b - 2(c - d)\}.$$

$$(28) a + 2b - 6a - \{3b - (6a - 6b)\}.$$

$$(29) 7a - \{3a - [4a - (5a - 2a)]\}.$$

$$(30) 2x - [3y - \{4x - (5y - 6x)\}].$$

(12) From $2a + b - c$, subtract $a - b$.

$$(13) \text{ ,, } 3ac - 2b, \text{ ,, } ac - b - d.$$

$$(14) \text{ ,, } 5ab - 6, \text{ ,, } -2ab + 6$$

$$(15) \text{ ,, } 4y^2 - 3y + 4, \text{ ,, } 2y^2 + 2y + 4$$

$$(16) 219a^3 - 117a^2b + 218ab^2 + 145b^3,$$
$$-(26a^3 + 4a^2b + 61ab^2 - 10b^3)$$

$$(17) a - x + 2y - 3z + w,$$

$$-(2x + 3a - y + z - w)$$

$$(18) 5x^3 + x^2y - 6xy^2 + y^3,$$
$$-(3x^3 + 4x^2y - 7xy^2 + y^3 - xy^3)$$

$$(19) y^4 - 4xy^3 + 7x^2y^2 - x^3y + 3x^4,$$
$$-(2x^4 + 3x^3y + x^2y^2 + xy^3)$$

$$(20) 2px^2 + ny^2 - 8qxy,$$
$$-(px^2 - 4qxy + 2ny^2)$$

$$(21) 2x^3 - 3x^2y + 2xy^2 - (x^3 - xy^2 + y^3).$$

$$(22) 7x^2 - xyz + 18z,$$
$$-(-3x^2 - 2xyz - p - q^3).$$

$$(73) (a^2 + b^2 + c^2 - ab - ac - bc) \\ (a + b + c).$$

$$(74) (a^4 + a^3b + a^2b^2 + ab^3 + b^4)(a - b).$$

$$(75) (2a + bc - 2b^2)(2a - bc + 2b^2).$$

$$(76) \text{ Multiply } 4ab - 2ac, \text{ by } \\ bab + 3ac.$$

$$(77) \text{ " } a + bx, \text{ by } a + cx.$$

$$(78) (x - 70)(x + 7)(x + 4).$$

$$(79) (x - 5)(x - 6)(x - 7)(x + 8).$$

$$(80) (a^3 + 3a^2b + 3ab^2 + b^3) \\ (a^3 - 3a^2b + 3ab^2 - b^3).$$

$$(81) (x^2 - a^2)(x^2 - xa + a^2)(x^2 + xa + a^2).$$

$$(82) (x^n - 2ax^{n-1}y + y^2) \times 3xy^n.$$

$$(83) (a^n + 2a^m b^n + ab^n)(a^m - b^n).$$

$$(84) (x^n + y^n)(x^n + yn).$$

$$(85) (x^m - y^m)(x^m + y^m)(x^n - y^n).$$

$$(86) (x^2 - x + 1)(x^2 + x + 1)(x^4 - x^2 + 1).$$

Multiplication.

$$(1) \text{ Multiply } a + b, \text{ by } ab.$$

$$(2) \text{ " } x + y, \text{ " } yz.$$

$$(3) \text{ " } 2ab + 3b^2, \text{ by } ac.$$

$$(4) \text{ " } 7a^2c + 5c^2b, \text{ " } bcd.$$

$$(5) \text{ " } 6x^3y + x^2y^2, \text{ " } yz.$$

$$(6) \text{ " } x^2xy + y^2, \text{ " } x + y.$$

$$(7) \text{ " } x^2 - xy + y^2, \text{ " } x^2 + xy + y^2.$$

$$(8) \text{ " } 3x^2 - 2xy + 5, \\ \text{ by } x^2 + 2xy - 6.$$

$$(9) \text{ " } x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 \\ - xy^5 + y^6, \text{ by } x + y.$$

$$(10) \text{ " } x^4 - 2x^3y + 4x^2y^2 - 8xy^3 \\ + 7y^4, \text{ by } x + 2y.$$

$$(11) \text{ " } 27a^2 - 13ab + 5b^2, \\ \text{ by } 7a^2 + b^2.$$

$$(12) (a + 1)(b + 1)(c + 1).$$

Division.

- (1) Divide $6ab - 8ax + 4a^2y$, by $2a$.
 (2) " $70a^2x - 75x^2$, " $5x$.
 (3) " $5xy + 20x^2y - 45axy$, " $5xy$.
 (4) " $-9a^2bc - 72ab^2c$, " $-3abc$.
 (5) " $6a^2x^2y - 9axy^2$, " $-3xy$.
 (6) " $a^2 - b^2$, by $a + b$.
 (7) " $4x^2 - y^2$, " $2x - y$.
 (8) " $a^2 - 2ax + x^2$, " $a - x$.
 (9) " $a^2 + ab - ac - bc$, by $a - c$.
 (10) " $xy - y^2 - xz + yx$, " $y - x$.
 (11) " $a^2 + 4ax + 4x^2$, " $a + 2x$.
 (12) " $a^3 - 3a^2x + 3ax^2 - x^3$,
 by $a - x$.
 (13) " $a^3 + 5a^2x + 5ax^2 + x^3$,
 by $a + x$.
 (14) " $a^4 - b^4$, by $a^3 + a^2b + ab^2 + b^3$.

- (27) $(x^4 - ax^3 + bx^2 - cx + d)$
 $(x^4 + ax^3 - bx^2 + cx - d)$.
 (28) $(x^3 + 4x^2 + 5x - 24)(x^2 - 4x + 11)$.
 (29) $(x^3 - 4x^2 + 77x - 24)(x^2 + 4x + 5)$.
 (30) $(a^4 - 2a^3 + 3a^2 - 2a + 7)$
 $(a^4 + 2a^3 + 3a^2 + 2a + 7)$.

$$(26) a^5 + a^3b^2 + 2a^2b^3 - b^5 \div a^2 - ab + b^2$$

$$(27) x^3 + ax^2 + bx + c, \div x - r.$$

$$(28) 1 + 2x, \div 1 - 3x.$$

$$(29) 1 + 2x, \div 1 - x - x^2.$$

$$(30) 1, \div 1 - x.$$

$$(75) (a^4 - 4a^3y + 6a^2y^2 - 4ay^3 + y^4) \div (a^2 - 2ay + y^2).$$

$$(76) 72x^4 - 792 \div 3x - 6.$$

$$(77) x^6 - 3x^4y^2 + 3x^2y^4 - y^6, \div x^3 - 3x^2y + 3xy^2 - y^3$$

$$(78) x^{4n} + x^{2ny}y^{2n} + y^{4n} \div x^{2n} + x^{ny}y^n + y^{2n}.$$

$$(79) a^2 - b^2 + 2bc - c^2 \div a - b + c$$

$$(20) x^4 - 6x^2y^2 - 76xy^3 - 75y^4, \div x^2 + 2xy + 3y^2$$

$$(21) ax^3 - a^2x^2 - bx^2 + b^2, \div ax - b$$

$$(22) mp^2x^3 + mq^2x^2 - np^2x^2 - mr^2x - nq^2x + nr^2, \div mx - n.$$

$$(23) a^3x^2 - a^3x + a^2x^2 + 2a^2x - 2a^2 + 2ax + ax^2 - ax^3 - x^3, \div a^2x + 2a - x^2$$

$$(24) -2a^{-8}x^5 + 77a^{-4}x^6 - 5x^7 - 24a^4x^8, \div 2a^{-3}x^3 - 3a^4x^4$$

$$(25) a^3 - 3a^2x + x^3, \div a + x.$$

- (17) Factor $4x^4 - 4x^2y + y^2$
- (18) " $36x^2 - 24xy + 4y^2$
- (19) " $4x^2y^2 - 4xyz + x^2$
- (20) " $a^2 - b^2$
- (21) " $4x^2 - 9y^2$
- (22) " $a^2c^2 - b^2d^2$
- (23) " $9a^2x^2 - 16a^2y^2$
- (24) " $25a^4b^4x^4 - 4z^2$
- (25) " $49x^4 - 76y^2$
- (26) " $x^2 + 73x + 42$
- (27) " $x^2 + 2x - 75$
- (28) " $x^2 - 75x + 56$
- (29) " $x^2 - x - 72$
- (30) " $8a^3 - b^3$
- (31) " $a^3 + 64m^3$
- (32) " $16a^4 + 36a^2b^2 + 81b^4$
- (33) " $a^4b^4 - 81c^4$

Factoring.

- (1) Factor $7a^2bc^3 - 28abc$
- (2) " $4x^4y^2 - 2x^2y^2$
- (3) " $6x^2y^2 + 72xy^3$
- (4) " $2a^2b + abc - abd$
- (5) " $7x^3y^2 - 7x^2y^3 + 7x^2y^2z$
- (6) " $75a^2cd + 20acd^2 - 75acd^2$
- (7) " $a^2 + 2ab + b^2$
- (8) " $4x^2 + 72xy + 9y^2$
- (9) " $x^2 + 72x + 36$
- (10) " $4x^4 + 4x^2y + y^2$
- (11) " $4a^2b^2 + 12abc + 9c^2$
- (12) " $16a^4y^4 + 8a^2y^4z^2 + y^4z^4$
- (13) " $a^2 + 2ab + b^2$
- (14) " $a^2x^2 - 2acx + c^2$
- (15) " $4x^2 - 4xy + y^2$
- (16) " $9a^2b^2 - 24a^2bc + 16a^2c^2$

Greatest Common Divisor.

Find the greatest common divisors
of the following;

- (1) of $42abx, 70acx$.
 (2) " $56acdxy, 24afx^2y$.
 (3) " $2x^3 - 4x^2y + 2xy^2, 2x^3 - 2xy^2$.
 (4) " $3ax^2 + 3x^3, 2ay + 2xy$.
 (5) " $4a^2c - 4acx, 3a^2y - 3agx$.
 (6) " $x^3 - y^3, x^2 - y^2$.
 (7) " $4c^2 - 12cx + 9x^2, 4c^2 - 9x^2$.
 (8) " $4ax^3 - 4axy^2, 12a^2x^2 - 12a^2y^2$.
 (9) " $2a^3x + 4a^2bx + 2ab^2x,$
 $4a^2x^3 + 8abx^3 + 4b^2x^3$.
 (10) " $2x^2 + 5xy + 3y^2, 3x^2 + xy - 2y^2$.
 (11) " $x^3 - 5x^2 + 7x - 3, x^2 + x - 12$.
 (12) " $3a^2x^3 - 3a^2y^3, 6ax^2 - 6ay^2$.
 (13) " $a^2 - 4, a^2 + 4a + 4$.

- (34) Factor $9x^4y^2 + 24x^3y^3 + 16x^2y^4$.
 (35) " $4x^2 - 12xy + 9y^2$.
 (36) " $a^2b^2c^2 - c^2d^2$.
 (37) " $x^2 + 9x + 18$.
 (38) " $2a^2x^2 - 2b^2x^2$.
 (39) " $a^2 - b^2 + 2bc - c^2$.
 (40) " $a^4 - 9a^2b^2 - 6abc^2 - c^4$.
 (41) " $6x^2 + xy - 12y^2$.
 (42) " $a^3 + 2ab + b^2 - c^2$.
 (43) " $6a^3 - 3ab + 5ac + 2bc - 6c^2$.
 (44) " $x^5 + 32y^5$.
 (45) " $64x^6 - 729y^6$.

(28) of $3a^2 - 3b^2, 3a^2 + 6ab + 3b^2,$
 $3axy + 3bxy.$

(29) " $x^2 - 9, x^2 - 3x - 18,$
 $x^2 + 7x + 24.$

(30) " $x^2 - 3x - 28, x^2 - 17x + 28,$
 $x^2 - 75x + 56.$

(31) " $x^4 - 5x^2 + 6, x^4 - 7x^2 + 12,$
 $x^4 + 2x^2 - 75.$

(32) " $x^3 + 5x^2 + 7x + 3, x^3 + 3x^2 - x - 3,$
 $x^3 + x^2 - 5x + 3.$

(33) " $x^2 + x + 7, x^3 + 2x^2 + 2x + 7,$
 $x^3 - 1.$

(34) " $x^4 - x^2 + 2x - 7, x^3 - 2x^2 + 2x - 7,$
 $x^3 + 7.$

(35) " $x^2 + xz - y^2 - yz, x^2 + xy - yz - x^2,$
 $xy + yz - xz - x^2.$

(74) of $a^3 - ab^2, a^2 + 2ab + b^2.$

(15) " $x^5 - x^3b^2, x^4 - b^4.$

(16) " $x^2 + 2x - 3, x^2 + 5x + 6.$

(17) " $3x^2y + 3xy^2, 3x^2 + 6xy + 3y^2.$

(18) " $x^4 + ax^3 - a^3x - a^4, x^4 + x^2a^2 + a^4.$

(19) " $20x^4 + x^2 - 7, 25x^4 + 5x^3 - x - 7.$

(20) " $a^4 - 2a^2b^2 + b^4, a^3 - 3a^2b + 3ab^2 - b^3.$

(21) " $2a^2x^2, 4x^2y^2, 8x^3y.$

(22) " $x^2 + 5x + 4, x^2 + 2x - 8,$
 $x^2 + 7x + 12.$

(23) " $3a^nx^n, 6a^{2n}x^{n+1}, 27a^{3n}x^{2n}.$

(24) " $x^2a - x^2b, 2ya^2 - 2yb^2, a^2 - ab.$

(25) " $7a^2 + 7ab, 4ab + 4b^2,$
 $2ac + 2bc.$

(26) " $3x^2 - 6x, 2x^3 - 4x^2, x^2y - 2xy.$

(27) " $3x^2 + 6xy, 2xy + 4y^2,$
 $4xz + 8zy.$

(15) of x^2+5x+6 , x^2+2x-8 ,
 $x^2+7x+12$.

(16) " $x-7$, x^2-7 , x^2+4x-5 .

(17) " $10x(x+y)$, $8y(x-y)$, $5(x^2-y^2)$.

(18) " $18x^4(x-y)$, $25x^3(x-y)^2$, $72x^5(x-y)^3$.

(19) " x^3-1 , x^2+x-2 .

(20) " $6x^2-x-7$, $2x^2+3x-2$.

(21) " $a-x$, a^2-x^2 , a^3-x^3 .

(22) " $3x^2-7x+6$, $2x^2-7x+3$,
 $6x^2-7x+2$.

(23) " $3x^2-5x+2$, $4x^3-4x^2-x+1$
 $6x^2-x-2$.

(24) " $2x-7$, $4x^2-7$, $4x^2+7$.

(25) " x^2-4a^2 , $(x+2a)^3$, $(x-2a)^3$.

Least Common Multiple.

Find the least common multiples
of the following:

(1) of $6ab^2$, $78a^2b$.

(2) " $6a^2xy$, $8ax^2$, $12x^2y^2$.

(3) " a^2-b^2 , $a^2-2ab+b^2$.

(4) " $15x^2y^2$, $6x^3y$.

(5) " $3x^2yz$, $6xy^3$, $9xyz$.

(6) " $3ab^2$, $6ac^3$, $4c^3d$, b^2c^2 .

(7) " $ax-bx$, $ay-by$, x^2y^2 .

(8) " $a-b$, a^2-b^2 , $a^2-2ab+b^2$.

(9) " $8x^2(x-y)$, $75x^5(x-y)^2$, $72x^3(x^2-y^2)$.

(10) " $2a^2(a+x)$, $4ax(a-x)$.

(11) " a^3-x^3 , a^2-x^2 .

(12) " $2x-7$, $4x^2-7$.

(13) " $x^2+7x+12$, $x^2+8x+15$.

(14) " x^2-y^2 , x^3+y^3 , $x^2-2xy+y^2$.

Reduce the following fractions to a mixed quantity:

$$(2) \frac{x^2+x-4}{x+2} \quad (1) \frac{a^2+x^2}{a+x}$$

$$(4) \frac{x^2+a^2+3-2ax}{x-a} \quad (3) \frac{a^3+x^3}{a^2+2ax+x^2}$$

$$(5) \frac{30-17x-44x^2+32x^3}{75+17x-4x^2}$$

$$(7) \frac{x^2+3x-25}{x-4} \quad (6) \frac{4x-x^2+3-y}{4-x}$$

$$(8) \frac{2y^4+19y^2+35}{y^3-3y^2+7y-27}$$

Reduce the following mixed quantity to a fractional form:

$$(2) a + \frac{ax}{a-x} \quad (1) a + \frac{b}{c}$$

$$(4) a + \frac{ac+d}{c+d} \quad (3) 7 + \frac{c}{x-y}$$

$$(6) 7 + \frac{a^2+b^2-c^2}{2ab} \quad (5) x+y + \frac{x^2+y^2}{x-y}$$

$$(7) ab+cd + \frac{abc-c^2d-2cd^2}{c+2d}$$

Transformation of Fractions.

Reduce the following fractions to their lowest terms:

$$(2) \frac{14ax}{27ay}$$

$$(7) \frac{12a^2cd}{16abc}$$

$$(4) \frac{16abx^2}{24a^2b^2x}$$

$$(3) \frac{45x^3y^3z}{36abx^2y^2z}$$

$$(6) \frac{2(x^2-y^2)}{x^2-2xy+y^2}$$

$$(5) \frac{3a^2-3b^2}{4a-4b}$$

$$(8) \frac{x^2-a^2}{x^2+2ax+a^2}$$

$$(7) \frac{5(a^2-x^2)}{10(a-x)}$$

$$(10) \frac{3x^2-6x}{2xy-4y}$$

$$(9) \frac{3ax^2-3a^2x}{2x^2y-2axy}$$

$$(12) \frac{x^2+x-2}{2x^3-3x+1}$$

$$(11) \frac{x^2+2ax+a^2}{3(x^2-a^2)}$$

$$(73) \frac{x^2-9}{x^2-x-12}$$

$$(74) \frac{x^2-2ax+a^2}{7x^2y-70axy+3a^2y}$$

$$(75) \frac{72x^2-15xy+3y^2}{6x^3-6x^2y+2xy^2-2y^3}$$

$$(14) \frac{4}{c-x}, \frac{5}{x^2}, \frac{6}{x^3}.$$

$$(15) 4, \frac{5}{a^2-x^2}, \frac{6}{a^2+x^2}, \frac{7}{y}.$$

Reduce the following fractions to a least common denominator:

$$(2) \frac{b}{ac}, \frac{c}{ab}, \frac{a}{bc} \quad (7) \frac{a}{b}, \frac{c}{d}, \frac{d}{x}.$$

$$(4) \frac{xy}{x+y}, \frac{axy}{(x+y)^2} \quad (3) \frac{a}{x-y}, \frac{a}{x+y}.$$

$$(5) \frac{x+a}{b}, \frac{a}{b}, \frac{a-x}{a}.$$

$$(6) \frac{a}{a^2-x^2}, \frac{b}{a-x}, \frac{c}{a+x}.$$

$$(7) \frac{m}{4a(a+x)}, \frac{n}{4(a^2-x^2)}.$$

$$(8) \frac{3x}{4}, \frac{4}{b}, \frac{72x^2}{75}.$$

$$(9) \frac{2b}{75}, \frac{3c}{5}, \frac{4d}{25}.$$

$$(10) a, \frac{3b^2}{4}, \frac{5c^3}{b}.$$

$$(11) \frac{x}{1-x}, \frac{x^2}{(1-x)^2}, \frac{x^3}{(1-x)^3}.$$

$$(12) 3bx, \frac{a}{a+x}, \frac{b}{a^2-x^2}, \frac{c}{x}.$$

$$(13) \frac{cx}{a-x}, \frac{dx^2}{a+x}, \frac{x^3}{a+x}.$$

Subtraction of Fractions.

$$(1) \frac{a+2x}{a-2x} - \frac{a-2x}{a+2x}$$

$$(2) 4a + \frac{2a}{c} - (2a - \frac{a-3b}{c})$$

$$(3) \frac{5x+3y}{4} - \frac{x-2y}{5}$$

$$(4) \frac{a}{a-x} - \frac{x}{a+x}$$

$$(5) \frac{x+y}{x-y} - \frac{x-y}{x+y}$$

$$(6) a + \frac{a-x}{a(a+x)} - \frac{a+x}{a(a-x)}$$

$$(7) 3x + \frac{11x-10}{15} - (2x + \frac{3x-5}{7})$$

$$(8) \frac{1}{y-x} - \frac{1}{y^2-x^2}$$

$$(9) \frac{a}{a-x} + \frac{3a}{a+x} - \frac{2ax}{a^2-x^2}$$

$$(10) \frac{3a-4b}{7} - \frac{2a-b-c}{3} + \frac{15a-4c}{12}$$

$$\frac{a-4b}{21}$$

Addition of Fractions.

$$(1) a + \frac{7}{a} + \frac{1}{2b} + \frac{3x}{4a^2}$$

$$(2) \frac{2}{a^2b^3} + \frac{3}{a^3b^2} + \frac{4}{a^3b^3}$$

$$(3) \frac{2a}{3x^2} + \frac{a+2x}{4x} + \frac{a}{6x}$$

$$(4) \frac{a}{a+b} + \frac{b}{a-b}$$

$$(5) \frac{2x}{7-x^2} + \frac{1}{x+7}$$

$$(6) \frac{2}{(x-7)^3} + \frac{3}{(x-7)^2} + \frac{4}{x-7}$$

$$(7) \frac{a}{(7+a)(a+x)} + \frac{x}{(7-x)(a+x)}$$

$$(8) \frac{2}{4(7+a)} + \frac{1}{4(7-a)} + \frac{1}{2(7-a^2)}$$

$$(9) \frac{3x-4y}{7} + \frac{-2x+y+7}{3} + \frac{15x-4}{12}$$

$$(10) \frac{a^2-bc}{(a+b)(a+c)} + \frac{b^2-ac}{(b+c)(a+b)} + \frac{c^2-ab}{(a+c)(b+c)}$$

- (71) $3 + \frac{x}{4}, x + \frac{4}{x}$.
- (72) $\frac{(a+b)^2}{2x}, \frac{4x^2}{a+b}$.
- (73) $\frac{3x^2+7}{2}, \frac{2y}{3}$.
- (74) $\frac{(x-7)^2}{y^3}, \frac{(x+7)y^2}{x-7}$.
- (75) $m + \frac{7}{m} - 7, m + \frac{7}{m} + 7$.
- (76) $x - \frac{y^2}{x}, \frac{x}{y} + \frac{y}{x}$.
- (77) $\frac{x(a-x)}{a^2+2ax+x^2}, \frac{a(a+x)}{a^2-2ax+x^2}$.
- (78) $\frac{a^2-x^2}{a+b}, \frac{a^2-b^2}{x(a+x)}, a + \frac{ax}{a-x}$.
- (79) $x + \frac{2xy}{x-y}, x - \frac{2xy}{x+y}$.
- (20) $\frac{a^3-x^3}{a^3+x^3}, \frac{a^2-x^2}{a^2+x^2}, \frac{a-x}{a+x}, \frac{a^2-ax+x^2}{a^2+ax+x^2}$.
- (21) $x^2+x+7, \frac{7}{x^2} - \frac{7}{x} + 7$.
- (22) $x+7+\frac{7}{x}, x-7+\frac{7}{x}$.
- (23) $\frac{2a-b}{4a}, \frac{6a-2b}{b^2-2ab}$.

Multiplication of Fractions.

Find the products of the following fractions.

- (1) $\frac{7x}{5y}, \frac{3a}{4c}$.
- (2) $\frac{2x}{x-y}, \frac{x^2-y^2}{3}$.
- (3) $2\left(\frac{x+y}{x-y}\right), \frac{x^2-y^2}{x^2+2xy+y^2}$.
- (4) $\frac{3x^2y}{4a}, \frac{2a^2b}{c}$.
- (5) $\frac{7abf}{3cd}, \frac{4x^2y^2}{3ab^2}$.
- (6) $\frac{7x-6}{3}, \frac{2x}{5}$.
- (7) $\frac{2}{x-y}, \frac{x^2-y^2}{a}$.
- (8) $\frac{ab}{4-x}, \frac{3x^2}{a^2}$.
- (9) $\frac{x^2-4}{3}, \frac{4x}{x+2}$.
- (10) $a + \frac{b}{x}, b + \frac{a}{x}$.

Division of Fractions.

- (1) Divide $\frac{3a^2}{a^2-b^2}$ by $\frac{a}{a+b}$.
- (2) " $\frac{3x}{2x-2}$, " $\frac{2x}{x-7}$.
- (3) " $\frac{(x+y)^2}{x-y}$, " $\frac{x+y}{(x-y)^2}$.
- (4) " $x + \frac{x}{x-7}$, " $x - \frac{x}{x-7}$.
- (5) " $\frac{x^3-3x^2a+3xa^2-a^3}{x+a}$ by $\frac{x-a}{x+11}$.
- (6) " $\frac{x^4-y^4}{a^3+b^3}$, by $\frac{x-y}{a^2-ab+b^2}$.
- (7) " $x^2+2+\frac{7}{x}$, " $\frac{x}{a} + \frac{7}{ax}$.
- (8) " $\frac{2ax+x^2}{c^3-x^3}$, " $\frac{x}{c-x}$.
- (9) " $x+y+\frac{x^2}{y}$, " $x+y+\frac{y^2}{x}$.
- (10) " $\frac{x^2-9}{x^2+4x+4}$, " $\frac{x+3}{x+2}$.
- (11) " 7, by $\frac{7}{a} + \frac{7}{b} + \frac{7}{c}$.

- (24) $\frac{a}{a+b} + \frac{b}{a-b}$, $\frac{a}{a+c} - \frac{b}{b+c}$.
- (25) $\frac{7-x^2}{7+y}$, $\frac{7-y^2}{x+x^2}$, $7 + \frac{x}{7-x}$.
- (26) $\frac{a^4-b^4}{a^2-2ab+b^2}$, $\frac{a-b}{a^2+ab}$.
- (27) $\frac{x+y}{2y}$, $\frac{x+y}{x-y} - \frac{x-y}{x+y} - \frac{4y^2}{x^2-y^2}$.
- (28) $\frac{a^3-b^3}{a^3+b^3}$, $\frac{a+b}{a-b}$, $\left(\frac{a^2-ab+b^2}{a^2+ab+b^2}\right)^2$.
- (29) $\frac{x^2}{a^2} - \frac{x}{a} + 7$, $\frac{x^2}{a^2} + \frac{x}{a} + 7$.
- (30) $\frac{4(a^2-ab)}{b(a+b)^2}$, $\frac{a^2-b^2}{bab}$.

$$(7) \frac{3}{2y-3} - \frac{2y+15}{4y^2+9} - \frac{2}{2y+3}$$

$$(8) \frac{x}{1-x} - \frac{x^2}{(1-x)^2} + \frac{x^3}{(1-x)^3}$$

$$(9) \left(\frac{x+2y}{x+y} + \frac{x}{y} \right) \div \left(\frac{x+2y}{y} - \frac{x}{x+y} \right)$$

$$(10) \frac{x^2-9x+20}{x^2-6x} \times \frac{x^2-73x+42}{x^2-5x}$$

$$(11) \frac{x^{3n}}{x^{n-7}} - \frac{x^{2n}}{x^{n+7}} + \frac{1}{x^{n+7}} - \frac{1}{x^{n-7}}$$

$$(12) \frac{a^2-ax}{bc+bx} \times \frac{4(a+x)}{3(c-x)}$$

$$(13) \frac{a}{b} - \frac{a^2-b^2}{b^2x} + \frac{a(a^2-b^2)x^2}{b^2(b+ax)}$$

$$(14) \frac{a^4-2a^2x^2+x^4}{a^3x+ax^3} \div \left(\frac{a+x}{a} \times \frac{a-x}{x} \right)$$

$$(15) \left(a + \frac{2ax-7}{b} \right) \div \frac{x-a}{ax+7}$$

$$(12) \text{ Divide } a^4 - \frac{7}{a^4}, \text{ by } a - \frac{1}{a}.$$

$$(13) \text{ " } \frac{x^2}{y^3} + \frac{7}{x}, \text{ by } \frac{x}{y^2} + \frac{7}{x} - \frac{7}{y}.$$

$$(14) \text{ " } \frac{a}{a-b} + \frac{b}{a-b}, \text{ by } \frac{a}{a-b} - \frac{b}{a+b}$$

$$(15) \text{ " } \frac{x+2y}{x+y} + \frac{x}{y}, \text{ by } \frac{x+2y}{y} - \frac{x}{x+y}$$

Miscellaneous examples.

Simplify the following expressions.

$$(1) \frac{a^2+ab+b^2}{2} + \frac{a^2-ab+b^2}{2}$$

$$(2) \frac{7}{a-7} - \frac{2a}{a^2+7} + \frac{7}{a+7}$$

$$(3) \frac{y-7}{2} + \frac{y-2}{3} + \frac{y+7}{6}$$

$$(4) \frac{2x}{x^2-y^2} + \frac{7}{x+y} - \frac{7}{x-y}$$

$$(5) \frac{x-y}{y} + \frac{2x}{x-y} - \frac{x^3+x^2y}{x^2y-y^3}$$

$$(6) \left(x + \frac{y-x}{7+xy} \right) \div \left(7 - x \frac{y-x}{7+xy} \right)$$

- (72) $\frac{b}{ax} - a^2 = b^2 - \frac{a}{bx}$.
- (73) $\frac{3x-4}{2} = \frac{x}{2} + \frac{x}{4} - \frac{7}{2}$.
- (74) $\frac{x}{8} - 1 + \frac{x}{12} - \frac{x+5}{4} = -\frac{77}{4}$.
- (75) $\frac{x+a}{b} - \frac{x}{a} = 1$.
- (76) $\frac{x-7}{2} + \frac{x-2}{3} - \frac{x-3}{4} = 6$.
- (77) $\frac{x}{3} - \frac{x}{4} - \frac{7}{2} = \frac{x}{5} - \frac{x}{6}$.
- (78) $\frac{3x-1}{7} + \frac{6-x}{4} - \frac{2x-4}{72} = \frac{54-x}{28}$.
- (79) $\frac{5x-7}{3} - \frac{3x-2}{7} = \frac{x-5}{4}$.
- (20) $\frac{x}{8} - \frac{2(x-7)}{5} = \frac{3x-4}{75} + \frac{x}{72}$.
- (21) $\frac{x-a}{3} - \frac{2x-3b}{5} - \frac{a-x}{2} = 70a + 11b$.
- (22) $\frac{6x+a}{4x+b} = \frac{3x-b}{2x-a}$.
- (23) $\frac{ax-b}{4} + \frac{a}{3} - \frac{bx}{2} - \frac{bx-a}{3}$.
- (24) $\frac{a+c}{a+x} + \frac{a-c}{a-x} = \frac{2b^2}{a^2-x^2}$.

Equations of the First Degree
with one unknown
quantity.

- (7) $\frac{x}{4} + \frac{2x}{3} = \frac{5}{6}$.
- (2) $\frac{2x}{5} - \frac{2x}{3} = \frac{5}{2}$.
- (3) $\frac{x}{7} + \frac{3x}{2} + \frac{x}{4} = 5$.
- (4) $\frac{73x}{72} + \frac{4}{3} = 6x, -\frac{79x}{4}$.
- (5) $\frac{x-4}{3} + \frac{x-2}{6} = \frac{5}{3}$.
- (6) $\frac{x-4}{3} - \frac{x-2}{6} = \frac{5}{3}$.
- (7) $\frac{x-3}{72} + \frac{3x-4}{27} = 8$.
- (8) $\frac{x}{3-x} + 4 = \frac{3}{5}$.
- (9) $73x + 76 = 7x + 20$.
- (70) $7x - 27 = -72x + 7$.
- (77) $5x - \frac{6x+3}{77} = \frac{7x+75}{2} - 3$.

$$(38) \frac{3x-7}{5} + \frac{25-4x}{9} = \frac{5x-74}{3}$$

$$(39) 79x + \frac{7}{2}(7x-2) = 4x + \frac{35}{2}$$

$$(40) x = 3x - \frac{7}{2}(4-x) + \frac{1}{3}$$

$$(41) \frac{2x+5}{73} + \frac{40-x}{8} = \frac{10x-427}{79}$$

$$(42) \frac{5x-7}{2} - \frac{2x+7}{3} = 3x-74$$

$$(43) \frac{x}{7} - \frac{x-5}{77} + 5 = x - \left(\frac{2x}{77} + 7\right)$$

$$(44) \frac{x-7}{2} + \frac{x-2}{3} = \frac{x+3}{4} + \frac{x+4}{6} + 7$$

$$(45) \frac{x-7}{x-2} - \frac{x-2}{x-3} = \frac{x-5}{x-6} - \frac{x-6}{x-7}$$

$$(46) (x-5)(x-2) - (x-5)(2x-5) \\ + (x+7)(x-2) = 0$$

$$(47) 3-x - 2(x-7)(x+2) \\ = (x-3)(5-2x)$$

$$(48) x-3 - (3-x)(x+7) \\ = (x-3)(7+x) + 3x$$

$$(25) \frac{6x+73}{75} - \frac{3x+5}{5} = \frac{2x}{5}$$

$$(26) \frac{x-3}{2} + \frac{x}{3} = 20 - \frac{x-79}{2}$$

$$(27) 10\left(x + \frac{7}{2}\right) - 6x\left(\frac{7}{x} - \frac{7}{3}\right) = 23$$

$$(28) \frac{x}{12} - \frac{8-x}{8} - \frac{5+x}{4} + \frac{71}{4} = 0$$

$$(29) 3x-4 - \frac{4}{5} \times \frac{7x-9}{3} = \frac{4}{5}\left(6 + \frac{x-7}{3}\right)$$

$$(30) \frac{4x}{5-x} - \frac{20-4x}{x} = \frac{75}{x}$$

$$(31) \frac{7x+5}{23} + \frac{9x-7}{70} - \frac{x-9}{5} + \frac{2x-3}{75} = 23\frac{1}{3}$$

$$(32) \frac{ax}{b} + \frac{cx}{d} + g = gx + \frac{cx}{d} + h$$

$$(33) \frac{70x+77}{78} - \frac{72x+2}{77x-8} = \frac{5x-4}{9}$$

$$(34) \frac{7}{7}\left(x - \frac{7}{2}\right) - \frac{7}{5}\left(\frac{2}{3} - x\right) = \frac{43}{30}$$

$$(35) \frac{2x+7}{29} - \frac{40x-3x}{72} = 9 - \frac{477-6x}{2}$$

$$(36) \frac{2x-7}{3} - \frac{3x-2}{4} = \frac{5x-4}{6} - \frac{7x+6}{72}$$

$$(37) \frac{2x-9}{27} + \frac{x}{18} - \frac{x-3}{4} = 8\frac{1}{3} - x$$

Equations of the First Degree
with two unknown
quantities.

$$(1) \frac{x}{3} + \frac{y}{5} = 5, \quad 2x + \frac{y}{3} = 17.$$

$$(2) 3x + 4y = 78, \quad 2x - y = 7.$$

$$(3) 7x - 3y = 72, \quad 2x + 2y = 72.$$

$$(4) 5x + 3y = 26, \quad 5x - y = 78.$$

$$(5) 4x + 3y = 76, \quad 3x + 4y = 79.$$

$$(6) 6x + y = 72, \quad x + 6y = 37.$$

$$(7) 4x + 5y = 77, \quad 3y - 2x = 8.$$

$$(8) x - 3y = 6, \quad 2x + 9y = 77.$$

$$(9) 2x - \frac{3}{4}y = 9, \quad x + y = 21.$$

$$(10) \frac{x}{2} - y = 7, \quad x - \frac{y}{2} = 8$$

$$(11) \frac{x+y}{70} + \frac{x-y}{2} = 0, \quad \frac{x+y}{5} + \frac{x-y}{2} = 1.$$

$$(12) \frac{2x-y}{4} - \frac{3}{2} = \frac{3y}{4} - x - 2,$$

$$\frac{x+y}{3} = 2\frac{2}{3}.$$

$$(49) \frac{x+10}{3} - \frac{3}{5}(3x-4) + \frac{(3x-2)(2x-3)}{6} = x^2 - \frac{8}{75}.$$

$$(50) (x+\frac{5}{2})(x-\frac{3}{2}) - (x+5)(x-3) + \frac{3}{4} = 0.$$

$$(25) \frac{2x+y}{9} + \frac{7x+6y+11}{18} = \frac{68-4x}{6},$$

$$\frac{21}{20} \left(\frac{x}{7} + \frac{y}{4} + \frac{4}{3} \right) = 4x - \frac{y}{8} - 24.$$

$$(26) \frac{x}{a} + \frac{y}{b} = 1, \quad \frac{x}{3a} + \frac{y}{6b} = \frac{2}{3}.$$

$$(27) \frac{7}{5}(2x+7y) - 7 = \frac{2}{3}(2x-6y+7),$$
$$x = 4y.$$

$$(28) x + \frac{7}{2}(3x-y-7) = \frac{7}{4} + \frac{3}{4}(y-7),$$
$$\frac{7}{5}(4x+3y) = \frac{7y}{70} + 2.$$

$$(29) ax+by=c, \quad mx-ny=d$$

$$(30) \frac{3x-5y}{2} + 3 = \frac{2x+y}{5},$$

$$8 - \frac{x-2y}{4} = \frac{x}{2} + \frac{y}{3}.$$

$$(73) \frac{x}{2} + \frac{y}{3} = 12, \quad \frac{x}{3} + \frac{y}{2} = 13.$$

$$(74) \frac{x+y}{2} - \frac{x-y}{3} = 8,$$

$$\frac{x+y}{3} + \frac{x-y}{4} = 11.$$

$$(75) 2x+3y=43, \quad 70x-y=7.$$

$$(76) 5x-7y=33, \quad 11x+12y=100.$$

$$(77) \frac{x}{2} + \frac{y}{3} = 1, \quad \frac{x}{3} + \frac{y}{4} = 7.$$

$$(78) 16x+17y=500, \quad 17x-3y=110.$$

$$(79) \frac{11x-5y}{22} = \frac{3x+y}{32}, \quad 8x-5y=7.$$

$$(20) 4x+8y=2.4, \quad 70.2x-6y=3.48.$$

$$(21) 13x+17y=4a, \quad 12x-6y=a.$$

$$(22) \frac{m}{x} + \frac{n}{y} = 1, \quad \frac{x}{m} + \frac{y}{n} = 1,$$

$$(23) 3.4x - .02y = .01,$$

$$2x + .4y = 1.2.$$

$$(24) \frac{x}{a} + \frac{y}{b} = 1 - \frac{x}{c}, \quad \frac{x}{b} + \frac{y}{a} = 1 + \frac{y}{c}.$$

$$(8) \frac{6y-4x}{3x-7} = 7, \quad \frac{5x-x}{2y-3x} = 7,$$

$$\frac{y-2x}{3y-2x} = 7.$$

$$(9) \frac{3}{x} - \frac{4}{5y} + \frac{7}{x} = \frac{38}{5}, \quad \frac{7}{3x} + \frac{7}{2y} + \frac{2}{x} = \frac{67}{6},$$

$$\frac{4}{5x} - \frac{7}{2y} + \frac{4}{x} = \frac{767}{70}.$$

$$(10) \frac{2}{x} - \frac{5}{3y} + \frac{7}{x} = \frac{85}{27}, \quad \frac{7}{4x} + \frac{7}{y} + \frac{2}{x} = \frac{443}{72},$$

$$\frac{5}{6x} - \frac{7}{y} + \frac{4}{x} = \frac{433}{36}.$$

$$(11) x-2y+3z=6, \quad 2x+3y-4z=20,$$

$$3x-2y+5z=26.$$

$$(12) 4x-3y+2z=40,$$

$$5x+9y-7z=47,$$

$$9x+8y-3z=97.$$

$$(13) 3x+2y+z=23,$$

$$5x+2y+4z=46,$$

$$70x+5y+4z=75.$$

Equations of the First Degree with
more than two unknown
quantities.

$$(1) 3x-4y+5z=74, \quad 3y+2z=70,$$

$$72x-8y-z=30.$$

$$(2) x+y+z=6, \quad 5x+2y-3z=0,$$

$$2x+y-z=7.$$

$$(3) 3x+2y-z=7, \quad x+y-z=7,$$

$$x+2y+3z=75.$$

$$(4) 2x-2y+3z=76, \quad 3x+5y-2z=6,$$

$$4x+3y-4z=-7.$$

$$(5) \frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 47, \quad \frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 38,$$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 62.$$

$$(6) \frac{x+y}{z} = 5, \quad \frac{y-z}{x} = 7, \quad \frac{x-z}{y} = \frac{7}{3}.$$

$$(7) x + \frac{1}{2}(y+z) = 102, \quad y + \frac{1}{3}(x+z) = 78,$$

$$z + \frac{1}{4}(x+y) = 61.$$

$$\begin{aligned}
 (20) \quad & 3x - 4y + 5z + 3v - 6u = 11, \\
 & 3x - 5y + 2z - 4u = 77, \\
 & 10y - 3z + 3u - 2v = 2, \\
 & 5z + 4u + 2v - 2x = 3, \\
 & 6u - 3v + 4x - 2y = 6.
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad & 5x - 6y + 4z = 75, \\
 & 7x + 4y - 3z = 19, \\
 & 2x + y + 6z = 46.
 \end{aligned}$$

$$\begin{aligned}
 (15) \quad & \frac{2}{x} + \frac{1}{y} = \frac{3}{z}, \quad \frac{3}{z} - \frac{2}{y} = 2, \\
 & \frac{1}{x} + \frac{1}{z} = \frac{4}{3}.
 \end{aligned}$$

$$(16) \quad \frac{3y-7}{4} = \frac{6z-x}{5} + 7\frac{4}{5},$$

$$\frac{5z}{4} + \frac{4z}{3} = y + \frac{5}{6},$$

$$\frac{3x+7}{7} - \frac{x}{74} + \frac{1}{6} = \frac{2z}{27} + \frac{y}{3}.$$

$$\begin{aligned}
 (17) \quad & 7x - 3y = 7, \quad 4z - 7y = 7, \\
 & 11x - 7u = 7, \quad 79x - 3u = 7.
 \end{aligned}$$

$$\begin{aligned}
 (18) \quad & 3u - 2y = 2, \quad 2x + 3y = 39, \\
 & 5x - 7z = 77, \quad 4y + 3z = 47.
 \end{aligned}$$

$$\begin{aligned}
 (19) \quad & 4u - 2x = 30, \quad 4y + 2z = 14, \\
 & 2x - 3y + 2z = 13, \\
 & 5y + 3u = 32.
 \end{aligned}$$

Evolution.

Extract the square roots of the following examples.

- (1) $a^4 - 2a^3 + 3a^2 - 2a + 1$.
- (2) $9x^2 - 30ax + 25a^2 + 5a^3 + \frac{a^4}{4} - 3a^2x$.
- (3) $4x^4 + 8ax^3 + 4a^2x^2 + 16b^2x^2 + 16ab^2x + 16b^4$.
- (4) $\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\left(\frac{a}{b} + \frac{b}{a}\right) + 3$.
- (5) $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} + \frac{xy}{3} + \frac{xz}{4} + \frac{yz}{6}$.
- (6) $\frac{x^2}{9} + \frac{4y^2}{25} + \frac{z^2}{16} + \frac{4xy}{75} + \frac{xz}{6} + \frac{yz}{5}$.
- (7) $x^4 + 2px^3 + (p^2 - 2q)x^2 - 2pqx + q^2$.
- (8) $(x + x^{-7})^2 - 4(x - x^{-7})$.
- (9) $4x^4 + 72x^3 + 5x^2 - 6x + 7$.
- (10) $4x^4 - 4x^3 + 5x^2 - 2x + 7$.
- (11) $4x^4 - 72ax^3 + 25a^2x^2 - 24a^3x + 16a^4$.
- (12) $25x^4 - 30ax^3 + 49a^2x^2 - 24a^3x + 16a^4$.

Involution.

- (2) $(-2axy^2)^3$
- (4) $(a^{-2})^2$
- (6) $(2x^2y^3)^{-2}$
- (8) $(ax^2y^3z^{-2})^3$
- (10) $(c + d)^4$
- (12) $(a + b)^6$
- (14) $(c - d)^4$
- (16) $(c - d)^6$
- (17) $(7 + 2x - 3x^2)^2$
- (18) $(a + b - c)^3$
- (19) $(7 + 2x + x^2)^3$
- (20) $(7 - 3x + 3x^2 - x^3)^2$
- (7) $(2a^2yx^3)^3$
- (3) $(-3a^2bc^3x)^4$
- (5) $(x^{-3}y)^{-2}$
- (7) $(2x^{-2}y^{-3})^{-2}$
- (9) $(a + b)^3$
- (11) $(a + b)^5$
- (13) $(a - b)^3$
- (15) $(a - b)^5$

$$(23) \quad 64a^6 - 288a^5 + 7080a^3 - 7458a - 729.$$

$$(24) \quad 7 - 6x + 27x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6.$$

$$(25) \quad a^3 + 3a^2b - 3a^2c + 3b^2a + 3c^2a + b^3 - 6abc - 3b^2c + 3c^2b - c^3.$$

$$(26) \quad 8x^3 + 36x^2y - 24x^2z + 54y^2x + 27z^3 - 72xyz + 24z^2x - 54y^2z + 36yz^2 - 8z^3.$$

$$(27) \quad 8x^6 - 36x^5 + 66x^4 - 63x^3 + 33x^2 - 9x + 7.$$

$$(28) \quad 8x^6 + 48x^5c + 60x^4c^2 - 80x^3c^3 - 90x^2c^4 + 708xc^5 - 27c^6.$$

$$(29) \quad 8x^6 - 36x^5c + 702x^4c^2 - 777x^3c^3 + 204x^2c^4 - 744xc^5 + 64c^6.$$

$$(30) \quad x^3 - \frac{7}{x^3} - 3x^2 - \frac{3}{x^2} + 5.$$

$$(73) \quad x^6 - 6x^5a + 75x^4a^2 - 20a^3x^3 + 75a^4x^2 - 6a^5x + a^6.$$

$$(74) \quad (a-b)^4 - 2(a^2+b^2)(a-b)^2 + 2(a^4+b^4).$$

$$(75) \quad 4\{(a^2-b^2)cd + ab(c^2-d^2)\}^2 + \{(a^2-b^2)(c^2-d^2) - 4abcd\}^2.$$

$$(16) \quad a^4 + b^4 + c^4 + d^4 - 2a^2(b^2+d^2) - 2b^2(c^2-d^2) + 2c^2(a^2-d^2).$$

$$(17) \quad 9a^{2m} + 6a^{3m+7} + 25c^{2m-4} + a^{4m+2} - 30a^m c^{m-2} - 70a^{2m+7} c^{m-2}.$$

$$(78) \quad 49x^4 + 9 - \frac{14x^3}{5} - \frac{6x}{5} + \frac{7057x^2}{25}.$$

$$(79) \quad 7 + x.$$

$$(20) \quad \frac{x^2}{y^2} \left(\frac{x^2}{4y^2} + 7 \right) + \frac{4y^2}{x^2} \left(\frac{y^2}{x^2} + 7 \right) + 3.$$

Extract the cube roots of the following examples.

$$(21) \quad 8x^3 - 72x^2 + 6x - 7.$$

$$(22) \quad x^6 - 6x^5 + 15x^4 - 20x^3 + 75x^2 - 6x + 7.$$

Theory of Indices.

- (1) Simplify $(x^{\frac{2}{3}} \times x^{\frac{4}{7}})^{\frac{74}{13}}$.
- (2) " $a^{\frac{7}{2}} \times a^{\frac{7}{3}} \times a^{\frac{7}{4}} \times a^{-\frac{7}{5}}$.
- (3) " $\left(\frac{ax}{x}\right)^{\frac{7}{2}} \times \left(\frac{bx}{y^2}\right)^{\frac{7}{3}} \times \left(\frac{y^2}{a^2b^2}\right)^{\frac{7}{4}}$.
- (4) " $a^{\frac{7}{3}} \times a^{\frac{3}{4}} \times \sqrt[3]{a^4} \times a^{\frac{7}{2}}$
 $\times \sqrt[8]{a^{\frac{25}{3}}} \times (a^{-\frac{2}{4}})^{\frac{7}{2}}$
- (5) $(a^{\frac{7}{2}} + b^{\frac{7}{2}} + a^{-\frac{7}{2}}b) \times (ab^{-\frac{7}{2}} - a^{\frac{7}{2}} + b^{\frac{7}{2}})$.
- (6) $(x^{\frac{3}{2}} - xy^{\frac{7}{2}} + x^{\frac{7}{2}}y - y^{\frac{3}{2}}) \times (x + x^{\frac{7}{2}}y^{\frac{7}{2}} + y)$.
- (7) $(a^{\frac{7}{2}} - a^3 + a^{\frac{5}{2}} - a^2 + a^{\frac{3}{2}} - a + a^{\frac{7}{2}} - 7)$
 $\times (a^{\frac{7}{2}} + 1)$
- (8) $(a^{\frac{2}{3}} - a^{\frac{7}{3}} + 1 - a^{-\frac{2}{3}} + a^{-\frac{2}{3}}) \times (a^{\frac{7}{3}} + 1 + a^{-\frac{7}{3}})$.
- (9) $(-3a^{-5} + 2a^{-4}b^{-1}) \times (-2a^{-3} - 3a^{-4}b)$.
- (10) $(x^{\frac{3}{2}} - xy^{\frac{7}{2}} + x^{\frac{7}{2}}y - y^{\frac{3}{2}}) \div (x^{\frac{7}{2}} - y^{\frac{7}{2}})$.
- (11) $(x^{\frac{4}{3}} + x^{\frac{2}{3}}y^{\frac{2}{3}} + y^{\frac{4}{3}}) \div (x^{\frac{2}{3}} + x^{\frac{7}{3}}y^{\frac{7}{3}} + y^{\frac{2}{3}})$.
- (12) $(a^{\frac{3n}{2}} - a^{-\frac{3n}{2}}) \div (a^{\frac{n}{2}} - a^{-\frac{n}{2}})$.

Transformation of Radicals.

Reduce the following radicals to a common index.

- (1) $2, 3^{\frac{7}{3}}, a^{\frac{7}{2}}, b^{\frac{7}{4}}$.
- (2) $a^{\frac{1}{2}}, b^2, c^{\frac{2}{3}}, d^{\frac{2}{3}}$.
- (3) $\sqrt{a+x}, \sqrt[3]{a-x}, \sqrt[4]{a^2-x^2}$.
- (4) $\sqrt{\frac{8}{3}}, \sqrt[3]{2}, 5\sqrt{3}$.
- (5) $ax, \sqrt{bx}, \sqrt[3]{cx}, \sqrt[4]{dx}$.
- (6) $cx^2, (dx^3)^{\frac{1}{2}}, (x^4)^{\frac{7}{2}}$.
- (7) $\sqrt[3]{7}, \sqrt{70}, \sqrt[6]{\frac{767}{49}}$.
- (8) $\sqrt{\frac{1}{2}}, \sqrt[4]{\frac{76}{49}}, \sqrt[6]{7337}$.
- (9) $\sqrt[3]{6}, \sqrt[4]{367}, \sqrt[6]{189}, \sqrt[8]{14647}$.
- (10) $\sqrt[5]{32}, \sqrt[3]{32}, \sqrt[4]{256}, \sqrt[6]{289}$.

Miscellaneous examples.

- (1) $\sqrt{18} + \sqrt{32} + \sqrt{50} + \sqrt{72}$.
- (2) $2\sqrt{8} + 3\sqrt{50} + 6\sqrt{18}$.
- (3) $\sqrt{\frac{3}{5}} + \sqrt{\frac{7}{15}} + \sqrt{\frac{15}{49}}$.
- (4) $\frac{2}{3}\sqrt{\frac{3}{9}} + \frac{7}{6}\sqrt{\frac{3}{36}} + \frac{3}{5}\sqrt{\frac{3}{32}}$.
- (5) $x\sqrt{(12a^4x)} - 2a^2\sqrt{(27x^3)} + 3a\sqrt{(48a^2x^3)}$
 $+ 5a^2x\sqrt{(3x)}$.
- (6) $6\sqrt[6]{(4a^2)} + 2\sqrt[3]{(2a)} + \sqrt[9]{(8a^3)}$.
- (7) $2\sqrt{3} + \frac{1}{2}\sqrt{12} + 4\sqrt{27} + 2\sqrt{\frac{3}{16}}$.
- (8) $3b\sqrt[3]{(2a^5b^2)} + 7\sqrt[3]{(2a^5b^5)} + 8a\sqrt[3]{(2a^2b^5)}$.
- (9) $\sqrt{320} - \sqrt{80}$.
- (10) $b\sqrt[3]{(27a^6b)} - \sqrt[3]{(216a^6b^4)}$.
- (11) $\sqrt{(a^3 + 2a^2b + ab^2)} - \sqrt{(a^3 - 2a^2b + ab^2)}$.
- (12) $\frac{2}{3}\sqrt{\frac{3}{9}} + \frac{3}{5}\sqrt{\frac{3}{32}} - \frac{7}{6}\sqrt{\frac{3}{36}}$.
- (13) $\sqrt{(289a^2b)} - \sqrt{(144a^2b)}$.
- (14) $(2\sqrt{8a^3} + 5\sqrt{72a^3}) - (7a\sqrt{18a} + \sqrt{50ab^2})$.

(13) $(2x^5y^{-3} - 5x^4y^{-2} + 7x^3y^{-1} - 5x^2 + 2xy)$
 $\div (x^3y^{-3} - x^2y^{-2} + xy^{-1})$.

(14) $(x^{\frac{5}{2}} - a^{\frac{3}{2}}b + ab^{\frac{3}{2}} - 2a^{\frac{7}{2}}b^2 + b^{\frac{5}{2}})$
 $\div (a^{\frac{3}{2}} - ab^{\frac{7}{2}} + a^{\frac{7}{2}}b - b^{\frac{3}{2}})$.

(15) Simplify
 $\frac{a^{\frac{3}{2}} - ax^{\frac{1}{2}} + a^{\frac{7}{2}}x - x^{\frac{3}{2}}}{a^{\frac{5}{2}} - a^2x^{\frac{1}{2}} - 3a^{\frac{3}{2}}x - 3ax^{\frac{3}{2}} + a^{\frac{7}{2}}x^2 - x^{\frac{5}{2}}}$.

$$(29) \left(\frac{x}{b} \sqrt{\frac{a}{b}} + \sqrt{\frac{c}{d}} \right) \times \left(\frac{x}{b} \sqrt{\frac{a}{b}} - \sqrt{\frac{c}{d}} \right).$$

$$(30) \left(\sqrt[3]{a^{-\frac{7}{2}}} + \sqrt[6]{a^{\frac{7}{2}}b} \right) \times \left(\sqrt[3]{a^{-\frac{7}{2}}} - \sqrt[6]{a^{\frac{7}{2}}b} \right).$$

$$(31) \frac{7}{3} \sqrt[3]{\frac{2}{3}} \div \frac{73}{4} \sqrt[3]{\frac{7}{5}}.$$

$$(32) \frac{7}{4} \sqrt{\frac{2}{5}} \div \frac{3}{7} \sqrt{\frac{5}{2}}.$$

$$(33) \frac{7}{2} \sqrt{2ax} \div \frac{3}{4} \sqrt{2bx}.$$

$$(34) \frac{7}{2} \sqrt{\frac{2}{3}} \div \frac{7}{3} \sqrt[3]{\frac{7}{3}}.$$

$$(35) 2\sqrt{2ax} \div \sqrt[3]{4bx^2}.$$

$$(36) x + \sqrt{xy} + y \div \sqrt{x + \sqrt[4]{xy} + \sqrt{y}}.$$

$$(37) \left(16x - \frac{y^4}{16} \right) \div \left(2x^{\frac{1}{4}} - \frac{y}{2} \right).$$

Render the denominators of the following fractions rational:

$$(38) \frac{-2}{\sqrt{3} + \sqrt{4}}.$$

$$(39) \frac{3}{\sqrt{2} - \sqrt{3}}.$$

$$(15) (a-x)\sqrt{(a^2-x^2)} - \sqrt{\frac{a+x}{a-x}}.$$

$$(16) (\sqrt[3]{87} + \sqrt[3]{192}) - \sqrt[3]{512}.$$

$$(17) 3\sqrt{8} \times 4\sqrt{48}.$$

$$(18) \frac{7}{3} \sqrt[3]{\frac{2}{3}} \times \frac{73}{4} \sqrt[3]{\frac{3}{4}}.$$

$$(19) 4\sqrt{72} \times 3\sqrt{2}.$$

$$(20) \frac{7}{3} \sqrt[3]{\frac{3}{4}} \times \frac{3}{4} \sqrt[3]{72}.$$

$$(21) 5a\sqrt{(ax)} \times \frac{5}{2} \sqrt{(bx)}.$$

$$(22) \sqrt{(2ab^3)} \times \sqrt{(8a^3b)}.$$

$$(23) \sqrt{8} \times \sqrt[3]{5}.$$

$$(24) \sqrt[3]{\left(\frac{1}{2}\right)} \times \sqrt{\left(\frac{3}{4}\right)}.$$

$$(25) \frac{7}{8} \sqrt[3]{\left(\frac{7}{8}\right)} \times \frac{7}{6} \sqrt[3]{\left(\frac{7}{8}\right)}.$$

$$(26) (\sqrt[3]{x} + 2\sqrt[6]{x} + 4) \times (\sqrt[3]{x} + 2\sqrt[6]{x}).$$

$$(27) (a^{\frac{3}{4}} + a^{\frac{1}{2}}x^{\frac{1}{2}} + a^{\frac{1}{4}}x + x^{\frac{3}{2}}) \times (a^{\frac{1}{4}} - x^{\frac{1}{2}}).$$

$$(28) \left(x + \frac{p}{2} + \sqrt{q + \frac{p^2}{4}} \right) \times \left(x + \frac{p}{2} - \sqrt{q + \frac{p^2}{4}} \right).$$

- (54) of $78 + 8\sqrt{5}$.
 (55) " $75 - 72\sqrt{21}$.
 (56) " $76 + 5\sqrt{7}$.
 (57) " $ab + c^2 + \sqrt{\{(a^2 - c^2)(b^2 - c^2)\}}$.
 (58) " $-9 + 6\sqrt{3}$.
 (59) " $7 + (7 - c^2)^{\frac{1}{2}}$.
 (60) " $6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}$.
 (61) " $5 + \sqrt{10} - \sqrt{6} - \sqrt{15}$.
 (62) " $15 - 2\sqrt{3} - 2\sqrt{5} + 6\sqrt{2} - 2\sqrt{6}$
 $+ 2\sqrt{5} - 2\sqrt{30}$.

- (63) " $17 + 2\sqrt{3} + 2\sqrt{7} + 2\sqrt{21}$.
 (64) " $11 + 4\sqrt{2} - 4\sqrt{5} - 2\sqrt{10}$.

Extract the cube roots:

- (65) of $7 + 5\sqrt{2}$.
 (66) " $76 + 8\sqrt{5}$.
 (67) " $9\sqrt{3} - 77\sqrt{2}$.
 (68) " $75\sqrt{6} - 21\sqrt{3}$.

(40) $\frac{4}{77 - 2\sqrt{3}}$.

(41) $\frac{3}{8 + \sqrt{2}}$.

(42) $(\sqrt{3} - \sqrt{2}) \div (\sqrt{2} + 1)$.

(43) $4 \div (\sqrt{5} + 1)$.

(44) $(\sqrt{a+x} + \sqrt{a-x}) \div (\sqrt{a+x} - \sqrt{a-x})$.

(45) $\frac{(3 + \sqrt{3})(3 + \sqrt{5})(\sqrt{5} - 2)}{(5 - \sqrt{5})(7 + \sqrt{3})}$.

Extract the square roots:

(46) of $14 + 6\sqrt{5}$.

(47) " $78 - 2\sqrt{77}$.

(48) " $94 + 42\sqrt{5}$.

(49) " $28 + 10\sqrt{3}$.

(50) " $(a+b)^2 - 4(a-b)\sqrt{ab}$.

(51) " $4 + 2\sqrt{3}$.

(52) " $7 - 4\sqrt{3}$.

(53) " $7 + 2\sqrt{10}$.

Radical Equations.

- (1) $\sqrt{x+16} = 2 + \sqrt{x}$.
- (2) $1 - \sqrt{7-x} = n(1 + \sqrt{7-x})$.
- (3) $\sqrt{a+x} - \sqrt{a-x} = \sqrt{ax}$.
- (4) $\frac{x-7}{\sqrt{x+7}} = 4 + \frac{\sqrt{x-7}}{2}$.
- (5) $\frac{x}{\sqrt{a^2+x^2}} = \frac{c-x}{\sqrt{b^2+(c-x)^2}}$.
- (6) $\frac{x-a}{\sqrt{x+a}} = \frac{\sqrt{x}-\sqrt{a}}{3} + 2\sqrt{a}$.
- (7) $8\sqrt{3x} + \frac{87(3+4\sqrt{3x})}{76x-3} = 76x+3$.
- (8) $\sqrt{x} = 7 + \sqrt{x-9}$.
- (9) $\sqrt{x} + \sqrt{x-3} = 3$.
- (10) $\sqrt{x} - \sqrt{2} = \sqrt{x-2}$.
- (11) $\sqrt[3]{4x+3} = 3$.
- (12) $\sqrt{5x+4} = 2 + \sqrt{3x}$.
- (13) $2\sqrt{x} - \sqrt{a} = 2\sqrt{x-a}$.
- (14) $a+x = \sqrt{x^2+5x-a}$.

(69) of $27\sqrt{6} - 23\sqrt{5}$.

(70) Simplify $\sqrt[3]{(\sqrt{5}+2)} - \sqrt[3]{(\sqrt{5}-2)}$.

Equations of the Second Degree with
but one unknown quantity.

$$(1) 3x^2 - 74x + 75 = 0.$$

$$(2) 4x - \frac{74-x}{x+7} = 74.$$

$$(3) \frac{3x+4}{5} - \frac{30-2x}{x-6} = \frac{7x-74}{70}.$$

$$(4) 5x^2 - 6x - 60 = 3.$$

$$(5) (x-72)(x+2) = 0.$$

$$(6) ax^2 - bx = c.$$

$$(7) \frac{70}{x} - \frac{74-2x}{x^2} = \frac{22}{9}.$$

$$(8) (x+2)^2 = 2x^2 + 8.$$

$$(9) 4x^2 - 9x = 90.$$

$$(10) \frac{x-3}{x+5} - \frac{x+4}{x-7} = 2\frac{7}{9}.$$

$$(11) x^2 - (a+b)x + ab = 0.$$

$$(12) \frac{4x^2}{3} = \frac{x}{3} + 11.$$

$$(15) \sqrt{a-x} = \frac{a}{\sqrt{a-x}} - x.$$

$$(16) \frac{\sqrt{x-2}}{3} + 3 = \frac{x-4}{\sqrt{x+2}}.$$

$$(17) x - \sqrt{a^2 + x\sqrt{x^2-1}} = a.$$

$$(18) \sqrt{x+a} = \sqrt{a} + \sqrt{x-a}.$$

$$(19) \sqrt{x} - \sqrt{a-x} = \frac{\sqrt{x} + \sqrt{a-x}}{2}.$$

$$(20) \sqrt[3]{7+x} + \sqrt[3]{7-x} = \sqrt[3]{2}.$$

$$(26) x^6 - 4x^3 = 32.$$

$$(27) x^4 - 2x^2 = 3.$$

$$(28) x^4 - 8x^2 = 9.$$

$$(29) x^3 - \sqrt{x^3} = 56.$$

$$(30) x^6 + 20x^3 = 69.$$

$$(31) x + \sqrt{70x + 6} = 9.$$

$$(32) \frac{\sqrt{4x+20}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$$

$$(33) 4x + 4\sqrt{x+2} = 7.$$

$$(34) x \pm \sqrt{5x+70} = 8.$$

$$(35) ax + 2\sqrt{n^2x + nax^2} = (3x-1)n.$$

$$(36) x^4 - 74x^2 = -1225.$$

$$(37) \frac{5x}{27}(x+7) - \frac{1}{2}(2x^2+x-7) = \frac{4}{35}(x+7).$$

$$(38) 8x + 77 + \frac{7}{x} = \frac{21+65x}{7}.$$

$$(39) \frac{6}{x} + \frac{x}{6} = \frac{5(x-7)}{4}.$$

$$(40) \frac{x}{7} + \frac{21}{x+5} = 3\frac{2}{7}.$$

$$(73) \frac{x}{x+7} + \frac{x+7}{x} = 2\frac{7}{6}.$$

$$(74) \frac{x+4}{3} - \frac{4x+7}{9} = \frac{7-x}{x-3} - 7.$$

$$(75) (x-7)^2 = 2(x^2+7).$$

$$(76) x^2(7 - \frac{7}{x}) = 8(x+2).$$

$$(77) 77x^2 + 79x - 7848 = 0.$$

$$(78) \frac{7}{2}x^2 - \frac{7}{3}x + 7\frac{3}{8} = 8.$$

$$(79) \frac{2x-70}{8-x} - \frac{x+3}{x-2} = 2.$$

$$(20) \frac{7}{x-7} - \frac{7}{x+3} = \frac{7}{35}.$$

$$(27) x + \frac{24}{x-7} = 3x-4.$$

$$(22) \frac{x^2+7}{2x} + \frac{x-7}{4} = 3x-2.$$

$$(23) x + \frac{7}{x} + 3(\frac{x-7}{4}) = \frac{7}{x}.$$

$$(24) x^3 + (5-x)^3 = 35.$$

$$(25) \frac{7200}{x} = \frac{7200}{40+x} + 5.$$

$$(53) x^4 - 74x^2 + 40 = 0.$$

$$(54) 2x + \sqrt{4x+8} = \frac{7}{2}.$$

$$(55) 2\sqrt{x} + \frac{2}{\sqrt{x}} = 5.$$

$$(56) \sqrt[4]{x} + 5\sqrt{x} - 22 = 0.$$

$$(57) 3\sqrt{x^3} - 4\sqrt[4]{x^3} = 7.$$

$$(58) x + 5 - \sqrt{x+5} = 6.$$

$$(59) 2(x^{\frac{1}{2}} + x^{-\frac{1}{2}}) = 5.$$

$$(60) \sqrt{2x+7} + \sqrt{3x-78} = \sqrt{7x+1}$$

$$(61) \frac{\sqrt{x^2-76}}{\sqrt{x-3}} + \sqrt{x+3} = \frac{7}{\sqrt{x-3}}.$$

$$(62) \sqrt{a+x} + \sqrt{a-x} = \sqrt{b}.$$

$$(63) \sqrt{x+9} = 2\sqrt{x} - 3.$$

$$(64) x + \sqrt{5x+10} = 8.$$

$$(65) (a^{\frac{1}{2}} + x^{\frac{1}{2}})^{\frac{2}{3}} = (a^{\frac{1}{3}} + x^{\frac{1}{3}})^{\frac{2}{3}}.$$

$$(66) \frac{\sqrt{a+x}}{\sqrt{a} + \sqrt{a+x}} = \frac{\sqrt{a-x}}{\sqrt{a} - \sqrt{a-x}}.$$

$$(67) \left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2 = n(n-1).$$

$$(41) \frac{27}{5-x} - \frac{x}{7} = 3\frac{2}{7}.$$

$$(42) \frac{3}{2(x^2-7)} + \frac{x}{4(x+7)} = \frac{3}{8}.$$

$$(43) \frac{1}{2(x-7)} + \frac{3}{x^2-7} = \frac{7}{4}.$$

$$(44) \frac{x}{75} + \frac{40}{3(70-x)} = \frac{3(70+x)}{95}.$$

$$(45) \frac{2x}{75} + \frac{3x-50}{3(70+x)} = \frac{72x+70}{790}.$$

$$(46) \frac{x+2}{x-7} - \frac{4-x}{2x} = \frac{7}{8}.$$

$$(47) \frac{x^2-5x}{x+3} = x-3 + \frac{7}{x}.$$

$$(48) \frac{x-6}{x-12} - \frac{x-72}{x-6} = \frac{5}{6}.$$

$$(49) \frac{x+4}{x-4} + \frac{x-4}{x+4} = \frac{70}{9}.$$

$$(50) (7-4\sqrt{3})x^2 + (2-\sqrt{3})x - 2$$

$$(51) \sqrt[3]{x} + \frac{5}{2\sqrt[3]{x}} = 3\frac{7}{4}.$$

$$(52) \sqrt{2x} - 7x = -52.$$

$$(81) \frac{a^2+x^2}{a+x} + \frac{a^2-x^2}{a-x} = 4a.$$

$$(82) \frac{\sqrt{x^2+7} + \sqrt{x^2-7}}{\sqrt{x^2+7} - \sqrt{x^2-7}} + \frac{\sqrt{x^2+7} - \sqrt{x^2-7}}{\sqrt{x^2+7} + \sqrt{x^2-7}} = 4\sqrt{x^2-7}.$$

$$(83) \sqrt{7-x+x^2} - \sqrt{7+x+x^2} = m.$$

$$(84) \frac{x + \sqrt{x^2-7}}{x - \sqrt{x^2-7}} + \frac{x - \sqrt{x^2-7}}{x + \sqrt{x^2-7}} = 34.$$

$$(85) \sqrt{x^2 - 3ax + a^2} + \sqrt{x^2 + 3ax + a^2} = \sqrt{2a^2 + 2x^2}.$$

$$(86) x\sqrt{\frac{6}{x} - x} = \frac{71x^2}{\sqrt{x}}.$$

$$(87) \sqrt[2p]{x^{p+q}} - \frac{7}{2c} \left(\sqrt[p]{x} + \sqrt[q]{x} \right) = 0.$$

$$(88) \sqrt{x} + \sqrt{x - \sqrt{7-x}} = 1.$$

$$(89) (x+a)^5 - (x-a)^5 = 242a^5.$$

$$(90) \frac{x^3+7}{x^2-7} = x + \sqrt{\frac{6}{x}}.$$

$$(91) \frac{25x^2 - 76}{10x + 8} = \frac{3(x^2 - 4)}{2x - 4}.$$

$$(68) (a+b)\sqrt{a^2+b^2+x^2} - (a-b)\sqrt{a^2+b^2-x^2} = a^2+b^2.$$

$$(69) x + \sqrt{x} + \sqrt{x+2} + \sqrt{x^2+2x} = a.$$

$$(70) 2x + \sqrt{2+2x} = c(1-x).$$

$$(71) \frac{a-x}{\sqrt{a} + \sqrt{a-x}} + \frac{a+x}{\sqrt{a} + \sqrt{a+x}} = \sqrt{a}.$$

$$(72) \frac{\sqrt{x+2a} - \sqrt{x-2a}}{\sqrt{x-2a} + \sqrt{x+2a}} = \frac{x}{2a}.$$

$$(73) \sqrt{x+8} - \sqrt{x+3} = \sqrt{x}.$$

$$(74) \sqrt{x+3} + \sqrt{x+8} = 5\sqrt{x}.$$

$$(75) \frac{x^2-a^2}{x^2+a^2} + \frac{x^2+a^2}{x^2-a^2} = \frac{34}{75}.$$

$$(76) \sqrt{x^2+a} - \sqrt{a} = c\sqrt{x^2+b}.$$

$$(77) \sqrt{x+4} - \sqrt{x} = \sqrt{x+\frac{3}{2}}.$$

$$(78) x^2 + \frac{7}{x^2} - a^2 - \frac{7}{a^2} = 0.$$

$$(79) \frac{850}{937} = \frac{x^2(x^4-a^4)}{x^6-a^6}.$$

$$(80) \frac{x^3-4x}{x-2} + \frac{x^2-7}{x+7} = 39.$$

General Properties of Equations
of the Second Degree.

Resolve the following five quadratic
expressions into the product of simple
factors.

(1) $3x^2 - 10x - 25$

(2) $x^2 + 73x + 780$

(3) $2x^2 + x - 6$

(4) $x^2 - 88x + 7612$

(5) $5x^2 - 3x - 110$

Form the quadratic equations,

(6) whose roots are 6 and 8.

(7) whose roots are 4 and 5.

(8) whose roots are 7 and -2.

(9) whose roots are $1 \pm \sqrt{5}$.

(92) $\sqrt{(x^2 + ax + b^2)} + \sqrt{(x^2 + bx + a^2)} = a + b$.

(93) $\sqrt{(2x+9)} + \sqrt{(3x-15)} = \sqrt{(7x+8)}$.

(94) $\sqrt{\frac{x}{a}} + \sqrt{\frac{(b-c)(ac-bx)}{abc}} = 1$.

(95) $\sqrt{(x^2 + 2x - 7)} + \sqrt{(x^2 + x + 7)} = \sqrt{2} + \sqrt{3}$.

(96) $\sqrt{(x^2 + ax - 1)} + \sqrt{(x^2 + bx - 1)} = \sqrt{a} + \sqrt{b}$.

(97) $\frac{x}{2} + \frac{\sqrt{(x-1)^3}}{\sqrt{(4x-1)}} = \frac{11}{16}$.

(98) $(x^2 + 7)(x + 2) = 2$.

(99) $(x-a)(x-b)(x-c) + abc = 0$.

(100) $\frac{1}{x+a+b} + \frac{1}{x-a+b} + \frac{1}{x+a-b}$
 $+ \frac{1}{x-a-b} = 0$.

Equations of the Second Degree
involving several unknown
quantities.

- (1) $x^2 + 72xy + y^2 = 85, x + 3y = 77.$
(2) $x^2 + y^2 = 202, x + y = 20.$
(3) $x^2 + y^2 = 394, x - y = 2.$
(4) $x^2 - 2xy + y^2 = 9, x + y = 77.$
(5) $x + y = 6, x^2 + y^2 = 26.$
(6) $x^2 - y^2 = 16, x + y = 8.$
(7) $x^2 + xy = 10, y^2 - xy = 15.$
(8) $x^2 + y^2 = 67, x^2 - xy = 6.$
(9) $x^2 + xy + y^2 = 37, x^2 - xy + y^2 = 73.$
(10) $x^2 - 2xy = 5, x^2 + y^2 = 29.$
(11) $3x^2 = 2xy + 24, y^2 = xy - 3.$
(12) $4xy - 3y^2 = 64, 2xy + 2x^2 - y^2 = 138.$
(13) $x^2 + y^2 + x + y = 922, \sqrt{xy} = 20.$

- (10) whose roots are 2 and -3.
(11) whose roots are 5 and $-\frac{2}{3}$.
(12) whose roots are a and b.
(13) whose roots are $\frac{3}{7}$ and $\frac{7}{3}$.
(14) whose roots are -7 and -3.
(15) whose roots are $-\frac{3}{4}$ and $\frac{5}{2}$.

$$(26) \quad x - \frac{x-y}{2} = 4, \quad y - \frac{x+3y}{x+2} = 1.$$

$$(27) \quad x^2 + y^2 = 65, \quad xy = 28.$$

$$(28) \quad xy = 1, \quad 3x - 5y = 2.$$

$$(29) \quad \frac{1}{x} + \frac{1}{y} = 2, \quad x + y = 2.$$

$$(30) \quad x^2 + xy + 2y^2 = 74, \\ 2x^2 + 2xy + y^2 = 73.$$

$$(31) \quad 2x + 3y = 37, \quad \frac{1}{x} + \frac{1}{y} = \frac{14}{45}.$$

$$(32) \quad x^2 + 3xy = 54, \quad xy + 4y^2 = 115.$$

$$(33) \quad x^2 + xy = 15, \quad xy - y^2 = 2.$$

$$(34) \quad x^2 + xy + 4y^2 = 6, \quad 3x^2 + 8y^2 = 14.$$

$$(35) \quad x^2 + xy = 12, \quad xy - 2y^2 = 1.$$

$$(36) \quad x^2 - xy + y^2 = 27, \quad y^2 - 2xy + 15 = 0.$$

$$(37) \quad x^2 - 4y^2 = 9, \quad xy + 2y^2 = 3.$$

$$(38) \quad 7x^2 - 8xy = 159, \quad 5x + 2y = 7.$$

$$(39) \quad x^2 - 2xy - y^2 = 1, \quad x + y = 2.$$

$$(74) \quad x + y + \sqrt{(x+y)} = 72, \quad x^3 + y^3 = 789.$$

$$(75) \quad x - y = 2, \quad x^4 + y^4 = 272.$$

$$(76) \quad x^2 + 2xy + y + 3x = 73, \\ y^2 + x + 3y = 44.$$

$$(77) \quad 3x^2 - 7y^2 + 7 = 0, \quad xy = 6.$$

$$(78) \quad x^4 - 2x^2y + y^2 = 49, \\ x^4 - 2x^2y^2 + y^4 - x^2 + y^2 = 20.$$

$$(79) \quad \frac{x}{y} - \frac{y}{x} = \frac{17}{30}, \quad x^2 + xy = 66.$$

$$(80) \quad x^2y^4 + y^2 = 70, \quad xy^2 + y = 4.$$

$$(21) \quad x^3 + y^3 = 789, \quad x^2y + xy^2 = 780.$$

$$(22) \quad \frac{x+y}{x-y} = a^2, \quad x^2 - y^2 = b^2.$$

$$(23) \quad 9x^2 = 4y^2, \quad 3xy + 2x + y = 485.$$

$$(24) \quad x^2 + y^2 - x - y = 78, \\ xy + x + y = 39.$$

$$(25) \quad \frac{1}{y} - \frac{1}{x} = \frac{1}{4}, \quad x^2y - xy^2 = 76.$$

$$(52) \quad xy(x+y) = 30, \quad x^3 + y^3 = 35.$$

$$(53) \quad \frac{x^2}{y} + \frac{y^2}{x} = 18, \quad x+y = 12.$$

$$(54) \quad x + \sqrt{(x^2 - y^2)} = 8, \quad x - y = 7.$$

$$(55) \quad x^2(x+y) = 80, \quad x^2(2x-3y) = 80.$$

$$(56) \quad x^2y + y^2x = 20, \quad \frac{1}{x} + \frac{1}{y} = \frac{5}{4}.$$

$$(57) \quad x^2 + y^2 = 7 + xy, \quad x^3 + y^3 = 6xy - 1.$$

$$(58) \quad x^2 + y^2 = 8, \quad \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}.$$

$$(59) \quad x+y = 4, \quad x^4 + y^4 = 82.$$

$$(60) \quad x^5 - y^5 = 3093, \quad x - y = 3.$$

$$(61) \quad \left(3 - \frac{6y}{x+y}\right)^2 + \left(3 + \frac{6y}{x-y}\right)^2 = 82, \quad xy = 2.$$

$$(62) \quad x^2 - x^2y^2 + y^2 = 19, \quad x - xy + y = 4.$$

$$(63) \quad x^2 - xy + y^2 = 7, \quad x^4 + x^2y^2 + y^4 = 133.$$

$$(64) \quad x^2 + xy + y^2 = 49, \quad x^4 + x^2y^2 + y^4 = 931.$$

$$(65) \quad x^4 - x^2 + y^4 - y^2 = 84, \\ x^2 + x^2y^2 + y^2 = 49.$$

$$(40) \quad \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{10}{3}, \quad x^2 + y^2 = 45.$$

$$(41) \quad \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2}, \quad x^2 + y^2 = 20.$$

$$(42) \quad .3x + .125y = 3x - y, \\ 3x - .5y = 2.25xy + 3y.$$

$$(43) \quad .1y + .125x = y - x, \\ y - .5x = .75xy - 3x.$$

$$(44) \quad y^2 - 4xy + 20x^2 + 3y - 264x = 0, \\ 5y^2 - 38xy + x^2 - 12y + 1056x = 0.$$

$$(45) \quad x+y = x^2, \quad 3y - x = y^2.$$

$$(46) \quad x^2 + y^2 = \frac{5}{2}xy, \quad x - y = \frac{1}{4}xy.$$

$$(47) \quad x + 2y + \frac{3x}{y} = 16, \quad 3x + y + \frac{3x}{y} = 23.$$

$$(48) \quad 4(x+y) = 3xy, \quad x+y+x^2+y^2 = 26.$$

$$(49) \quad x - y = 2, \quad x^3 - y^3 = 8.$$

$$(50) \quad x + y = 5, \quad x^3 + y^3 = 65.$$

$$(51) \quad x + y = 11, \quad x^3 + y^3 = 7007.$$

$$(75) \sqrt{x+y} + 2\sqrt{x-y} = \frac{2(x-7)}{\sqrt{x-y}},$$

$$\frac{x^2+y^2}{xy} = \frac{34}{75}.$$

$$(76) yz = bc, \quad \frac{x}{a} + \frac{y}{b} = 1, \quad \frac{x}{a} + \frac{z}{c} = 1.$$

$$(77) \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9, \quad \frac{2}{x} + \frac{3}{y} = 13,$$

$$8x + 3y = 5.$$

$$(78) x(x+y+z) = 24, \quad y(x+y+z) = 28,$$

$$z(x+y+z) = 12.$$

$$(79) \frac{1}{29} \left(x + \frac{y}{z}\right) = \frac{1}{6}, \quad \frac{1}{34} \left(y + \frac{x}{z}\right) = \frac{1}{6},$$

$$x + y + z = 15.$$

$$(80) xy + xz + yz = 26,$$

$$xy(x+y) + yz(y+z) + xz(z+x) = 162,$$

$$xy(x^2+y^2) + yz(y^2+z^2) + xz(x^2+z^2) = 538.$$

$$(66) x(12-xy) = y(xy-3),$$

$$xy(y+4x-xy) = 12(x+y-3).$$

$$(67) x+y+\sqrt{xy} = 14,$$

$$x^2+y^2+xy = 84.$$

$$(68) x+y-\sqrt{xy} = 7,$$

$$x^2+y^2+xy = 133.$$

$$(69) x+y = 72, \quad \sqrt[3]{x} + \sqrt[3]{y} = 6.$$

$$(70) \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \sqrt{\frac{7}{xy}} + 1,$$

$$\sqrt{(x^3y)} + \sqrt{(y^3x)} = 78.$$

$$(71) x+y = 10, \quad \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{5}{2}.$$

$$(72) \sqrt{3+x^2} + 2y = 8,$$

$$2x^2 + \sqrt{5y^2+4x^4} = 9.$$

$$(73) \frac{x}{a} + \frac{y}{b} = 1, \quad \frac{a}{x} + \frac{b}{y} = 4.$$

$$(74) \sqrt{(x^2+y^2)} + \sqrt{(x^2-y^2)} = 2y,$$

$$x^4 - y^4 = a^4.$$

Answers.
Explanation of signs.

- | | | |
|------------|----------------------|------------------------|
| (1) 78. | (2) 8. | (3) 14. |
| (4) 42. | (5) 39. | (6) 9. |
| (7) 7. | (8) 9. | (9) 54. |
| (10) 78. | (11) 3. | (12) 4. |
| (13) 23. | (14) 6. | (15) 22. |
| (16) 75. | (17) 0. | (18) 10. |
| (19) 32. | (20) 4. | (21) 75. |
| (22) 1. | (23) 17088. | (24) $14\frac{1}{2}$. |
| (25) 3465. | (26) $7\frac{7}{10}$ | (27) 4.6875. |
| (28) 75. | (29) $2\frac{2}{3}$ | (30) 268. |

Addition.

- (1) $16a^2bc$ (2) $-14a^2bc$ (3) $4a^2bc$
(4) $-2a^2bc$ (5) $77a - 12bx$
(6) $10c + 2bx^2 - d$
(7) $14x^2y - y^2x - 7y + z$

Answers.

- (26) $-8b - 109e + 37f - 10g + h.$
 (27) $6a^2b + 5abc - 3b^2c - 14c^3 + 2cd^2 - 3d^3.$
 (28) $10a^2bc + bx + 15af.$
 (29) $a^2n^2 - 3a^3m + 6b + 5mn.$
 (30) $5a^3b^2c + 5ax^3d + 25a^4x.$

Subtraction.

- (1) $3a^2b^2c.$ (2) $4a^2b.$ (3) $4a^2bc.$
 (4) $6ab.$ (5) $71ac$ (6) $13b^2d.$
 (7) $-11a^2bc.$ (8) $2a^4b^n.$ (9) $3a^2 - 3b.$
 (10) $-2x^2 + 2x^2y + 11.$
 (11) $7xy^2 - 3x + 6x^3.$ (12) $a + 2b - c.$
 (13) $2ac - b + d.$ (14) $7ab - 12.$
 (15) $2y^2 - 5y.$
 (16) $193a^3 + 157ab^2 - 121a^2b + 155b^3.$
 (17) $-2a - 3x + 3y - 4z + 2w.$ (8)
 (18) $2x^3 - 3x^2y + xy^2 + xy^3.$ (9)

Answers.

- (8) $9a + 8d$ (9) $5cx^2 + 2dy^2 + z^3 + d$
 (10) $5ab + 8c + 4(a + b)$
 (11) $3x^2y + 8(a + b)z^2$
 (12) $2a + 2b + 2c$
 (13) $20ax + 13by$ (14) $22a^2 + 7ab - 8b^2.$
 (15) $-2x^3 + 6x^2y + 10xy^2 - 2y^3.$
 (16) $73x + 73y + 28z + 72.$
 (17) $9x^3 + 2y^3 + 3xyz.$
 (18) $4x^4 + 14x^3y - 43x^2z + 21xv.$
 (19) $3ax + x - by - y.$
 (20) $7ax + 3bx + 6by - ay.$
 (21) $9px - 6qy + 2c.$
 (22) $3x^2 - a^2x - 5ax - x$
 (23) $-4a^2x - 2x^2 + ax.$
 (24) $-2a - 2x + 4y - z.$
 (25) $3ax^2y + bxy^2z^2 + dxy^2x^2 + 2cxz^3 + 4bdx.$

Answers.

- (11) $189a^4 - 91a^3b + 62a^2b^2 - 13ab^3 + 5b^4$
 (12) $x^3 + ax^2 + bx^2 + cx^2 + abx + acx + bcx + abc$
 (13) $a^3 + b^3 + c^3 - 3abc$. (14) $a^5 - b^5$
 (15) $4a^2 - b^2c^2 + 4b^3c - 4b^4$
 (16) $24a^2b^2 - 6a^2c^2$
 (17) $a^2 + abx + acx + bcx^2$
 (18) $x^3 - 5x^2 - 4bx - 40$
 (19) $x^4 - 10x^3 - 37x^2 + 64bx - 7680$
 (20) $a^6 - 3a^4b^2 + 3a^2b^4 - b^6$
 (21) $x^6 - ab$
 (22) $3x^{n+1}y^n - 6ax^ny^{n+1} + 3xy^{n+2}$
 (23) $a^{m+n} + 2a^2mb^n + a^{m+1}b^n - a^nb^n - 2a^mb^{2n} - ab^{n+n}$
 (24) $x^{2n} + 2x^ny^n + y^{2n}$
 (25) $x^{2m+n} - x^ny^{2m} - x^{2m}y^n + y^{2m+n}$

Answers.

- (19) $x^4 - 4x^3y + 6x^2y^2 - 5xy^3 + y^4$
 (20) $px^2 - 4qxy - 7y^2$
 (21) $x^3 + 3xy^2 - 3x^2y - y^3$
 (22) $10x^2 + xyz + 18x + p + q^3$
 (23) $x^3 - 3x^2y + 3xy^2 - y^3$
 (24) $2x + 2y + 2a$
 (25) $a^2 - 3b^2 + 3c^2$ (26) $2y + 2z$
 (27) $a - b + c - d$ (28) $a - 7b$
 (29) $5a$ (30) $12x - 8y$

Multiplication.

- (1) $a^2b + ab^2$ (2) $xyx + y^2x$
 (3) $2a^2b^2c + 3ab^3c$ (4) $7a^2c^2bd + 5c^3b^2d$
 (5) $6x^3y^2z + x^2y^3z$ (6) $x^3 + y^3$
 (7) $x^4 + x^2y^2 + y^4$
 (8) $3x^4 + 4x^3y - 4x^2y^2 - 13x^2 + 22xy - 30$
 (9) $x^7 + y^7$ (10) $x^5 + 32y^5$

Answers.

- (16) $4x^3 + 8x^2 + 16x + 32.$
 (17) $x^3 + 3x^2y + 3xy^2 + y^3$
 (18) $x^{2n} - x^n y^n + y^{2n}.$ (19) $a + b - c.$
 (20) $x^2 - 2xy - 5y^2.$ (21) $x^2 - ax - b.$
 (22) $px^2 + qx - r.$ (23) $ax - a + x.$
 (24) $-a^5x^2 + 7a^7x^3 + 8a^3x^4.$
 (25) $a^2 - 4ax + 4x^2 - \frac{3x^3}{a+x}.$
 (26) $a^3 + a^2b + ab^2 + 2b^3 + \frac{ab^4 - 3b^5}{a^2 - ab + b^2}.$
 (27) $x^2 + rx + ax + r^2 + ar + b + \frac{r^3 + ar^2 + b^2 + c}{x-r}.$
 (28) $1 + 5x + 15x^2 + 45x^3 + \dots$
 (29) $1 + 3x + 4x^2 + 7x^3 + \dots$
 (30) $1 - x + x^2 - x^3 + x^4, \dots$

Factoring.

- (1) $7abc(ac^2 - 4).$ (2) $2x^2y^2(2x^2 - 1).$
 (3) $6xy^2(x + 2y).$ (4) $ab(2a + c - d).$
 (5) $7x^2y^2(x - y + z).$ (6) $5acd(3a + 4c - 3d).$

Answers.

- (26) $x^8 + x^4 + 1.$
 (27) $x^8 - a^2x^6 - b^2x^4 - c^2x^2 - d^2 + 2abx^5$
 $- 2acx^4 + 2adx^3 + 2bcx^3 - 2bdx^2 +$
 $2cdx.$
 (28) $x^5 + 757x - 264.$
 (29) $x^5 - 47x - 720$
 (30) $a^8 + 2a^6 + 3a^4 + 2a^2 + 1.$

Division.

- (1) $3b - 4x + 2ay.$ (2) $2a^2 - 3x.$
 (3) $7 + 4x - 9a.$ (4) $3a + 4b.$
 (5) $-2a^2x + 3ay.$ (6) $a - b.$
 (7) $2x + y.$ (8) $a - x.$
 (9) $a + b.$ (10) $x - y.$
 (11) $a + 2x.$ (12) $a^2 - 2ax + x^2.$
 (13) $a^2 + 4ax + x^2.$ (14) $a - b.$
 (15) $a^2 - 2ay + y^2.$

Answers.

- (37) $(a+4m)(a^2-4am+16m^2)$.
 (38) $(4a^2+6ab+9b^2)(4a^2-6ab+9b^2)$.
 (39) $(a^2b^2+9c^2)(ab+3c)(ab-3c)$.
 (40) $(3x^2y+4xy^2)(3x^2y+4xy^2)$.
 (41) $(2x-3y)(2x-3y)$.
 (42) $c^2(ab+d)(ab-d)$.
 (43) $(x+6)(x+3)$. (38) $2x^2(a+b)(a-b)$.
 (44) $(a+b-c)(a-b+c)$.
 (45) $(a^2+3ab+c^2)(a^2-3ab-c^2)$.
 (46) $(3x-4y)(2x+3y)$.
 (47) $(a+b-c)(a+b+c)$.
 (48) $(2a-b+3c)(3a-2c)$.
 (49) $(x+2y)(x^4-2x^3y+4x^2y^2-8xy^3+16y^4)$.
 (50) $(2x-3y)(2x+3y)(4x^2-6xy+9y^2)$
 $(4x^2+6xy+9y^2)$.

Answers.

- (7) $(a+b)(a+b)$. (8) $(2x+3y)(2x+3y)$.
 (9) $(x+b)(x+b)$. (10) $(2x^2+y)(2x^2+y)$.
 (11) $(2ab+3c)(2ab+3c)$.
 (12) $(4a^2y^2+y^2z^2)(4a^2y^2+y^2z^2)$.
 (13) $(a-b)(a-b)$. (14) $(ax-c)(ax-c)$.
 (15) $(2x-y)(2x-y)$. (16) $(3ab-4ac)(3ab-4ac)$.
 (17) $(2x^2-y)(2x^2-y)$. (18) $(6x-2y)(6x-2y)$.
 (19) $(2xy-z)(2xy-z)$.
 (20) $(a+b)(a-b)$. (21) $(2x+3y)(2x-3y)$.
 (22) $(ac+bd)(ac-bd)$.
 (23) $(3ax+4ay)(3ax-4ay)$.
 (24) $(5a^2b^2x^2+2z)(5a^2b^2x^2-2z)$.
 (25) $(7x^2+4y)(7x^2-4y)$. (26) $(x+6)(x+7)$.
 (27) $(x-3)(x+5)$. (28) $(x-7)(x-8)$.
 (29) $(x+8)(x-9)$.
 (30) $(2a-b)(4a^2+2ab+b^2)$.

Answers.

- (31) $x^2 - 3$. (32) $x + 3$.
 (33) $x^2 + x + 7$. (34) $x^2 - x + 1$.
 (35) $x + y + z$.

Least common multiple.

- (1) $18a^2b^2$. (2) $24a^2x^2y^2$.
 (3) $a^3 - a^2b - ab^2 + b^3$. (4) $30x^3y^2$.
 (5) $18x^2y^3z$. (6) $12ab^2c^3d$.
 (7) $ax^2y^2 - bx^2y^2$. (8) $(a - b)^2(a + b)$.
 (9) $120x^5(x^2 - y^2)(x - y)$. (10) $4a^2x(a^2 - x^2)$.
 (11) $a^4 + a^3x - ax^3 - x^4$. (12) $4x^2 - 1$.
 (13) $(x + 3)(x + 4)(x + 5)$.
 (14) $x^5 - 2x^4y + x^3y^2 + x^2y^3 - 2xy^4 + y^5$.
 (15) $(x^2 + 2x - 8)(x^2 + 5x + 6)$.
 (16) $(x^2 - 1)(x + 5)$. (17) $40xy(x^2 - y^2)$.
 (18) $900x^5(x - y)^3$. (19) $x^4 + 2x^3 - x^2 - 2$.
 (20) $6x^3 + 11x^2 - 3x - 2$.

Answers.

Greatest common divisor.

- (1) $14ax$. (2) $8ax^2y$.
 (3) $2x(x - y)$. (4) $a + x$.
 (5) $a(a - x)$. (6) $x - y$.
 (7) $2c - 3x$. (8) $4ax^2 - 4ay^2$.
 (9) $2x(a + b)^2$. (10) $x + y$.
 (11) $x - 3$. (12) $3a(x - y)$.
 (13) $a + 2$. (14) $a + b$.
 (15) $x^2 - b^2$. (16) $x + 3$.
 (17) $8x + 3y$. (18) $x^2 + ax + a^2$.
 (19) $5x^2 - 7$. (20) $a^2 - 2ab + b^2$.
 (21) $2x^2$. (22) $x + 4$.
 (23) $3a^n - 1x^{n-1}$. (24) $a - b$.
 (25) $a + b$. (26) $x^2 - 2x$.
 (27) $x + 2y$. (28) $3(a + b)$.
 (29) $x + 3$. (30) $x - 7$.

Answers.

- (5) $2 - 3x + \frac{5x^2}{5-x}$ (6) $x + \frac{3-y}{4-x}$
 (7) $x + 7 + \frac{3}{x-4}$ (8) $2y + 6 + \frac{23}{y-3}$

Fractional form.

- (1) $\frac{ac+b}{c}$ (2) $\frac{a^2}{a-x}$ (3) $\frac{x-y+c}{x-y}$
 (4) $\frac{2ac+ad+d}{c+d}$ (5) $\frac{2x^2}{x-y}$
 (6) $\frac{(a+b)^2-c^2}{2ab}$ (7) $\frac{2ab(c+d)}{c+2d}$

Common denominator.

- (1) $\frac{adx}{bdx}, \frac{bcx}{bdx}, \frac{bd^2}{bdx}$ (2) $\frac{b^2}{abc}, \frac{c^2}{abc}, \frac{a^2}{abc}$
 (3) $\frac{a(x+y)}{x^2-y^2}, \frac{a(x-y)}{...}$ (4) $\frac{xy(x+y)}{(x+y)^2}, \frac{axy}{...}$
 (5) $\frac{a(x+a)}{ab}, \frac{a^2}{...}, \frac{b(a-x)}{...}$
 (6) $\frac{a}{a^2-x^2}, \frac{b(a+x)}{...}, \frac{c(a-x)}{...}$
 (7) $\frac{m(a-x)}{4a(a^2-x^2)}, \frac{na}{...}$

Answers.

- (21) $a^4 + a^3 - ax^3 - x^4$
 (22) $6x^3 - 25x^2 + 23x - 6$
 (23) $(3x-2)(4x^3 - 4x^2 - x + 1)$
 (24) $16x^4 - 7$ (25) $(x^2 - 4a^2)^3$

Transformation of fractions.

Lowest terms.

- (1) $\frac{3ad}{4b}$ (2) $\frac{2x}{3y}$ (3) $\frac{5xy}{4ab}$
 (4) $\frac{2x}{3ab}$ (5) $\frac{3(a+b)}{4}$ (6) $\frac{2(x+y)}{x-y}$
 (7) $\frac{7}{2}(a+x)$ (8) $\frac{x-a}{x+a}$ (9) $\frac{3a}{2y}$
 (10) $\frac{3x}{2y}$ (11) $\frac{x+a}{3(x-a)}$ (12) $\frac{x+2}{2x^2+2x-7}$
 (13) $\frac{x-3}{x-4}$ (14) $\frac{x-a}{y(7x-3a)}$ (15) $\frac{72x-3y}{6x^2+2y^2}$

Mixed quantity.

- (1) $a - x + \frac{2x^2}{a+x}$ (2) $x - 7 - \frac{2}{x+2}$
 (3) $a - 2x + \frac{3x^2}{a+x}$ (4) $x - a + \frac{3}{x-a}$

Answers.

(8) $\frac{7}{7-ax}$ (9) $\frac{85x-20y}{84}$ (10) 0.

Subtraction of fractions.

(1) $\frac{8ax}{a^2-4x^2}$ (2) $2a + \frac{3(a-b)}{c}$

(3) $\frac{27x+23y}{20}$ (4) $\frac{a^2+x^2}{a^2-x^2}$

(5) $\frac{4xy}{x^2-y^2}$ (6) $a - \frac{4x}{a^2-x^2}$

(7) $\frac{737x+5}{705}$ (8) $\frac{x+y-1}{y^2-x^2}$

(9) $\frac{4a}{a+x}$ (10) $\frac{81a-4b}{84}$

Multiplication of fractions.

(1) $\frac{27ax}{20cy}$ (2) $\frac{2x(x+y)}{3}$ (3) 2.

(4) $\frac{3abx^2y}{2c}$ (5) $\frac{28x^2y^2}{9bcd}$

(6) $\frac{74x^2-72x}{75}$ (7) $\frac{2(x+y)}{a}$ (8) $\frac{3bx^2}{a(4-x)}$

(9) $\frac{4x(x-2)}{3}$ (10) $\frac{(ax+b)(bx+a)}{x^2}$

(11) $\frac{(72+x)(x^2+4)}{4x}$ (12) $2x(a+b)$

Answers.

(8) $\frac{45x}{60}, \frac{40}{...}, \frac{48x^2}{...}$ (9) $\frac{70b}{75}, \frac{45c}{...}, \frac{12d}{...}$

(10) $\frac{72a}{72}, \frac{96^2}{...}, \frac{10c^3}{...}$

(11) $\frac{x(7-x)^2}{(7-x)^3}, \frac{x^2(7-x)}{...}, \frac{x^3}{...}$

(12) $\frac{3bx^2(a^2-x^2)}{x(a^2-x^2)}, \frac{ax(a-x)}{...}, \frac{bx}{...}, \frac{c(a^2-x^2)}{...}$

(13) $\frac{cx(a+x)}{a^2-x^2}, \frac{dx^2(a-x)}{...}, \frac{x^3(a-x)}{...}$

(14) $\frac{4x^3}{x^3(c-x)}, \frac{5x(c-x)}{...}, \frac{6(c-x)}{...}$

(15) $\frac{4y(a^4-x^4)}{y(a^4-x^4)}, \frac{5y(a^2+x^2)}{...}, \frac{6y(a^2-x^2)}{...}$

$\frac{7(a^4-x^4)}{...}$

Addition of fractions.

(1) $\frac{(2a^3+4a+3x)b+2a^2}{4a^2b}$

(2) $\frac{2a+3b+4}{a^3b^3}$ (3) $\frac{6x^2+5ax+8a}{72x^2}$

(4) $\frac{a^2+b^2}{a^2-b^2}$ (5) $\frac{1}{7-x}$

(6) $\frac{4x^2-5x+3}{(x-7)^3}$ (7) $\frac{1}{(7+a)(7-x)}$

Answers.

- (8) $\frac{2a+x}{c^2+cx+x^2}$. (9) $\frac{x}{y}$.
 (10) $\frac{x-3}{x+2}$. (11) $\frac{abc}{ab+ac+bc}$.
 (12) $a^3+a+\frac{1}{a}+\frac{1}{a^3}$. (13) $\frac{x+y}{y}$.
 (14) 1. (15) 1.

Miscellaneous examples.

- (1) a^2+b^2 . (2) $\frac{4a}{a^4-7}$. (3) y .
 (4) $\frac{2}{x+y}$. (5) $\frac{y}{x-y}$. (6) y .
 (7) $\frac{78(2y+75)}{16y^4-87}$. (8) $x+\frac{x^4}{(7-x)^3}$. (9) 1.
 (10) $\frac{(x-4)(x-7)}{x^2}$. (11) $x^{2m}+x$.
 (12) $\frac{4a(a^2-x^2)}{3b(c^2-x^2)}$. (13) $\frac{a+bx}{b+ax}$.
 (14) $\frac{a^2-x^2}{a^2+x^2}$. (15) $\frac{a^2x(b+2x)+a(x+b)-1}{b(x-a)}$.

Answers.

- (73) $\frac{2y(3x^2+7)}{b}$. (74) $\frac{x^2-7}{y}$.
 (75) $m^2+7+\frac{1}{m^2}$. (76) $\frac{x^4-y^4}{x^2y}$.
 (77) $\frac{ax}{a^2-x^2}$. (78) $\frac{a^2(a-b)}{x}$.
 (79) x^2 . (80) $\frac{(a-x)^3}{a^3+a^2x+ax^2+x^3}$.
 (81) $x^2+7+\frac{1}{x^2}$. (82) $x^2+7+\frac{1}{x^2}$.
 (83) $\frac{b-3a}{2ab}$. (84) $\frac{(a^2+b^2)c}{(a+b)(a+c)(b+c)}$.
 (85) $\frac{1-y}{x}$. (86) $\frac{a^2+b^2}{a}$.
 (87) 2. (88) $\frac{a^2-ab+b^2}{a^2+ab+b^2}$.
 (89) $\frac{x^4}{a^4}+\frac{x^2}{a^2}+7$. (90) $\frac{2(a-b)^2}{3b^2(a+b)}$.

Division of fractions.

- (1) $\frac{3a}{a-b}$. (2) $\frac{3}{4}$. (3) x^2-y^2 .
 (4) $\frac{x}{x-2}$. (5) $(x-a)^2$.
 (6) $\frac{x^3+x^2y+xy^2+y^3}{a+b}$. (7) $\frac{ax^2+a}{x}$.

Answers.

- | | | |
|-----------------------|-----------------------|-----------------------|
| (40) $\frac{2}{3}$. | (41) 56. | (42) 7. |
| (43) 7. | (44) $8\frac{3}{5}$. | (45) $4\frac{1}{2}$. |
| (46) $2\frac{3}{7}$. | (47) $1\frac{4}{7}$. | (48) 3. |
| (49) 2. | (50) 12. | |

Equations of the First Degree with two unknown quantities.

- | | |
|----------------------------|----------------------------|
| (1) $x=6, y=15$. | (2) $x=2, y=3$. |
| (3) $x=3, y=3$. | (4) $x=4, y=2$. |
| (5) $x=7, y=4$. | (6) $x=7, y=6$. |
| (7) $x=\frac{7}{2}, y=3$. | (8) $x=7, y=\frac{7}{3}$. |
| (9) $x=9, y=12$. | (10) $x=10, y=4$. |
| (11) $x=4, y=6$. | (12) $x=3, y=5$. |
| (13) $x=12, y=18$. | (14) $x=18, y=6$. |
| (15) $x=2, y=13$. | (16) $x=8, y=7$. |
| (17) $x=-6, y=12$. | (18) $x=10, y=20$. |
| (19) $x=7, y=11$. | (20) $x=.4, y=.1$. |

Answers.

Equations of the First Degree with one unknown quantity.

- | | | |
|-------------------------------|------------------------------|----------------------------|
| (1) $\frac{10}{77}$. | (2) $3\frac{1}{22}$. | (3) $2\frac{34}{53}$. |
| (4) 8. | (5) $6\frac{2}{3}$. | (6) 0. |
| (7) $37\frac{6}{19}$. | (8) $4\frac{1}{4}$. | (9) $\frac{2}{3}$. |
| (10) $1\frac{3}{19}$. | (11) 5. | (12) $\frac{7}{ab}$. |
| (13) 2. | (14) 12. | (15) $-a$. |
| (16) 77. | (17) 10. | (18) 5. |
| (19) $\frac{67}{83}$. | (20) $\frac{80}{67}$. | (21) $25a+24b$. |
| (22) $\frac{a^2-b^2}{b-4a}$. | (23) $\frac{3b}{3a-2b}$. | (24) $\frac{a^2-b^2}{c}$. |
| (25) $-\frac{2}{9}$. | (26) $23\frac{1}{4}$. | (27) 2. |
| (28) 72. | (29) $7\frac{1}{3}$. | (30) $3\frac{2}{77}$. |
| (31) 79. | (32) $\frac{b(h-g)}{a-bg}$. | (33) 4. |
| (34) $\frac{43}{9}$. | (35) 72. | (36) 4. |
| (37) 9. | (38) 4. | (39) 7. |

Answers.

- (4) $x = 3, y = 1, z = 4.$
 (5) $x = 24, y = 60, z = 120.$
 (6) $x = 4, y = 6, z = 2.$
 (7) $x = 62, y = 46, z = 34.$
 (8) $x = 10, y = 7, z = 3.$
 (9) $x = \frac{7}{2}, y = \frac{1}{3}, z = \frac{1}{4}.$
 (10) $x = 6, y = 9, z = \frac{1}{3}.$
 (11) $x = 8, y = 4, z = 2.$
 (12) $x = 10, y = 2, z = 3.$
 (13) $x = 4, y = 3, z = 5.$
 (14) $x = 3, y = 4, z = 6.$
 (15) $x = \frac{7}{6}, y = -\frac{7}{2}, z = \frac{21}{10}.$
 (16) $x = 2, y = 3, z = 1.$
 (17) $x = 4, y = 9, z = 16, u = 25.$
 (18) $u = 4, x = 12, y = 5, z = 7.$
 (19) $x = 3, y = 1, u = 9, z = 5.$

Answers.

- (21) $x = y = \frac{a}{b}.$ (22) $x = y = m + n.$
 (23) $x = .02, y = 2.9.$
 (24) $x = \frac{(ab + ac - bc)abc}{b^2a^2 + a^2c^2 - b^2c^2},$
 $y = \frac{(ac - ab - bc)abc}{b^2a^2 + a^2c^2 - b^2c^2}.$
 (25) $x = 7, y = 4.$
 (26) $x = 3a, y = -2b.$
 (27) $x = 4, y = 1.$ (28) $x = \frac{10}{3}, y = \frac{20}{3}.$
 (29) $x = \frac{nc + bd}{mb + na}, y = \frac{mo - ad}{mb + na}.$
 (30) $x = 12, y = 6.$

Equations of the First Degree with
 more than two unknown
 quantities.

- (1) $x = 4, y = 2, z = 2.$
 (2) $x = 7, y = 2, z = 3.$
 (3) $x = 2, y = 2, z = 3.$

Answers.

(16) $c^6 - 6c^5d + 15c^4d^2 - 20c^3d^3 + 15c^2d^4 - 6cd^5 + d^6$

(17) $1 + 4x + 70x^2 + 72x^3 + 9x^4$

(18) $a^3 + 3a^2b - 3a^2c + 3b^2a + 3c^2a + b^3 - 6abc - 3b^2c + 3c^2b - c^3$

(19) $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$

(20) $1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$

Square roots.

(1) $a^2 - a + 1$. (2) $3x - 5a - \frac{a^2}{2}$

(3) $2x^2 + 2ax + 4b^3$ (4) $\frac{a}{b} + \frac{b}{a} + 1$

(5) $\frac{x}{2} + \frac{y}{3} - \frac{z}{4}$. (6) $\frac{x}{3} + \frac{2y}{5} - \frac{z}{4}$

(7) $x^2 + px - q$. (8) $x - x^{-1} - 2$

(9) $2x^2 + 3x - 7$. (10) $2x^2 - x + 7$

(11) $2x^2 - 3ax + 4a^2$

(12) $5x^2 - 3ax + 4a^2$

Answers.

(20) $x=2, y=-1, z=3, u=-7, v=-2$.

Involution.

(1) $8a^6y^3x^9$. (2) $-8a^3x^3y^6$

(3) $81a^8b^4c^2x^4$ (4) a^{-4}

(5) x^6y^2 . (6) $\frac{7}{4}x^{-4}y^{-6}$

(7) $\frac{7}{4}x^4y^6$. (8) $a^{-3}x^{-6}y^{-9}z^6$

(9) $a^3 + 3a^2b + 3ab^2 + b^3$

(10) $c^4 + 4c^3d + 6c^2d^2 + 4cd^3 + d^4$

(11) $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

(12) $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

(13) $a^3 - 3a^2b + 3ab^2 - b^3$

(14) $c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4$

(15) $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

Answers.

- (3) $\sqrt[12]{(a+x)^6}$, $\sqrt[12]{(a-x)^4}$, $\sqrt[12]{(a^2-x^2)^3}$.
 (4) $2\sqrt[6]{\frac{8}{27}}$, $\sqrt[6]{4}$, $5\sqrt[6]{27}$.
 (5) $(a^{12}x^{12})^{\frac{1}{12}}$, $(b^6x^6)^{\frac{1}{12}}$, $(c^4x^4)^{\frac{1}{12}}$, $(d^3x^3)^{\frac{1}{12}}$.
 (6) $(cx^2)^{\frac{4}{4}}$, $(dx^3)^{\frac{1}{4}}$, $(24)^{\frac{2}{4}}$.
 (7) $\sqrt[6]{49}$, $\sqrt[6]{1000}$, $\sqrt[6]{\frac{161}{49}}$.
 (8) $\sqrt{\frac{1}{2}}$, $\sqrt{\frac{4}{7}}$, $\sqrt{11}$.
 (9) $\sqrt[6]{36}$, $\sqrt[6]{(19)^3}$, $\sqrt[6]{189}$, $\sqrt[6]{(11)^3}$.
 (10) $\sqrt[3]{8}$, $\sqrt[3]{32}$, $\sqrt[3]{64}$, $\sqrt[3]{17}$.

Theory of Indices.

- (1) $x^{\frac{4}{3}}$. (2) $a^{\frac{17}{60}}$. (3) $\frac{y^{\frac{1}{3}}}{(bx)^{\frac{1}{2}}}$. (4) 1.
 (5) $a^{\frac{3}{2}}b^{-\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}} + a^{-\frac{1}{2}}b^{\frac{3}{2}}$.
 (6) $x^{\frac{5}{2}} + x^{\frac{3}{2}}y - xy^{\frac{3}{2}} - y^{\frac{5}{2}}$. (7) $a^4 - 7$.
 (8) $a + a^{\frac{1}{3}} - 1 + a^{-\frac{1}{3}} + a^{-1}$.
 (9) $-4a^{-7}b^{-1} + 9a^{-9}b$. (10) $x + y$.

Answers.

- (13) $(x-a)^3$. (14) $a^2 + b^2$.
 (15) $(a^2+b^2)(c^2+d^2)$. (16) $a^2 - b^2 + c^2 - d^2$.
 (17) $3a^m - 5c^{m-2} + a^{2m+7}$.
 (18) $7x^2 - \frac{x}{5} + 3$.
 (19) $7 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{76} - 2c$.
 (20) $\frac{x^2}{2y^2} + \frac{2y^2}{x^2} + 7$.

Cube roots.

- (21) $2x - 7$. (22) $x^2 - 2x + 7$.
 (23) $4a^2 - 6a - 9$. (24) $1 - 2x + 3x^2$.
 (25) $a + b - c$. (26) $2x + 3y - 2z$.
 (27) $2x^2 - 3x + 7$. (28) $2x^2 + 4cx - 3c^2$.
 (29) $2x^2 - 3cx + 4c^2$. (30) $x - \frac{7}{x} - 7$.

Common index.

- (1) $(2^{12})^{\frac{1}{12}}$, $(3^4)^{\frac{1}{12}}$, $(a^6)^{\frac{1}{12}}$, $(b^3)^{\frac{1}{12}}$.
 (2) $(a^3)^{\frac{1}{6}}$, $(b^{12})^{\frac{1}{6}}$, $(c^4)^{\frac{1}{6}}$, $(d^4)^{\frac{1}{6}}$.

Answers.

- (23) $2\sqrt[6]{200}$. (24) $\sqrt[6]{\frac{27}{256}}$.
 (25) $\frac{7}{96}\sqrt[3]{\frac{7}{6}}$. (26) $\sqrt[3]{x^2+4}\sqrt{x+8}\sqrt[3]{x+8}\sqrt[6]{x}$.
 (27) $a-x^2$. (28) x^2+px-q .
 (29) $\frac{ax^2}{b^3}-\frac{c}{d}$. (30) $\sqrt[3]{a^7}-\sqrt[3]{(a^2b)}$.
 (31) $\frac{28}{39}\sqrt[3]{\frac{70}{3}}$. (32) $\frac{7}{30}$. (33) $\frac{2}{3}\sqrt{\frac{a}{b}}$.
 (34) $\frac{3}{2}\sqrt[6]{\frac{8}{3}}$. (35) $2\sqrt[6]{\frac{a^3}{2b^2x}}$.
 (36) $\sqrt{x}-\sqrt[4]{xy}+\sqrt{y}$.
 (37) $8x^{\frac{3}{4}}+2x^{\frac{7}{2}}y+\frac{7}{2}x^{\frac{7}{4}}y^2+\frac{7}{8}y^3$.
 (38) $-2(2-\sqrt{3})$. (39) $-3(\sqrt{2}+\sqrt{3})$.
 (40) $\frac{44+8\sqrt{3}}{709}$. (41) $\frac{24-3\sqrt{2}}{b^2}$.
 (42) $\sqrt{2}-\sqrt{3}+\sqrt{6}-2$. (43) $\sqrt{3}-7$.
 (44) $\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}$. (45) $\frac{7}{5}\sqrt{15}$.
 (46) $3+\sqrt{5}$. (47) $\sqrt{17}-\sqrt{7}$.

Answers.

- (11) $x^{\frac{2}{3}}-x^{\frac{7}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$. (12) $a^{n+1}+a^{-n}$.
 (13) $2x^2-3xy+2y^2$.
 (14) $a+a^{\frac{1}{2}}b^{\frac{1}{2}}-b$. (15) $\frac{x+a}{x^2+3xa+a^2}$.

Miscellaneous examples.

- (1) $18\sqrt{2}$. (2) $37\sqrt{2}$. (3) $\frac{4^3}{705}\sqrt{75}$.
 (4) $\frac{2}{5}\sqrt[3]{b}$. (5) $25a^2x\sqrt{3x}$.
 (6) $9\sqrt[3]{2a}$. (7) $\frac{37}{2}\sqrt{3}$.
 (8) $18ab\sqrt[3]{(2a^2b^2)}$. (9) $4\sqrt{5}$.
 (10) $-3a^2b\sqrt[3]{b}$. (11) $2b\sqrt{a}$.
 (12) $\frac{37}{90}\sqrt[3]{b}$. (13) $5a\sqrt{b}$.
 (14) $(73a-5b)\sqrt{2a}$.
 (15) $(a-x-\frac{1}{a-x})\sqrt{(a^2-x^2)}$. (16) $7\sqrt[3]{3}-8$.
 (17) $96\sqrt{6}$. (18) $\frac{97}{24}\sqrt[3]{4}$.
 (19) $24\sqrt{6}$. (20) $\frac{7}{2}\sqrt[3]{6}$.
 (21) $\frac{25ax}{2}\sqrt[3]{(ab)}$. (22) $4a^2b^2$.

Answers.

Radical Equations.

- (1) 9. (2) $\frac{4n}{(7+n)^2}$. (3) $\frac{4a^2}{a^2+4}$
 (4) 87. (5) $\frac{ac}{a+b}$. (6) $16a$.
 (7) 3. (8) 25. (9) 4.
 (10) 2. (11) 6. (12) 12 .
 (13) $\frac{25a}{7b}$. (14) $\frac{a^2+a}{5-2a}$. (15) $a-7$.
 (16) $42\frac{1}{4}$. (17) $\frac{4a^2+7}{4a}$. (18) $\frac{5a}{4}$.
 (19) $\frac{9a}{70}$. (20) ± 7 .

Equations of the Second Degree with
 but one unknown quantity.

- (1) 3, $\frac{5}{3}$. (2) 4, $-\frac{7}{2}$. (3) 36, 72 .
 (4) $\frac{27}{5}$, -3 . (5) 12, -2 .
 (6) $\frac{-b \pm \sqrt{b^2+4ac}}{2a}$. (7) 3, $\frac{21}{77}$.

Answers.

- (48) $7 + 3\sqrt{5}$. (49) $5 + \sqrt{3}$.
 (50) $a - 2\sqrt{ab} - b$. (51) $7 + \sqrt{3}$.
 (52) $2 - \sqrt{3}$. (53) $\sqrt{5} + \sqrt{2}$.
 (54) $\sqrt{70} + 2\sqrt{2}$. (55) $3\sqrt{7} - 2\sqrt{3}$.
 (56) $\sqrt{\frac{25}{2}} + \sqrt{\frac{7}{2}}$.
 (57) $\sqrt{\left\{\frac{(a+c)(b+c)}{2}\right\}} + \sqrt{\left\{\frac{(a-c)(b-c)}{2}\right\}}$.
 (58) $\sqrt[4]{3}\left(\frac{3}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}}\right)$.
 (59) $\sqrt[4]{\frac{7}{7-c^2}}\left\{\sqrt{\left(\frac{7+c}{2}\right)} + \sqrt{\left(\frac{7-c}{2}\right)}\right\}$.
 (60) $7 + \sqrt{2} + \sqrt{3}$. (61) $7 + \sqrt{\frac{5}{2}} - \sqrt{\frac{3}{2}}$.
 (62) $\sqrt{6} + \sqrt{3} - \sqrt{5} - 7$. (63) $7 + \sqrt{3} + \sqrt{7}$.
 (64) $2 + \sqrt{2} - \sqrt{5}$. (65) $7 + \sqrt{2}$.
 (66) $7 + \sqrt{5}$. (67) $\sqrt{3} - \sqrt{2}$.
 (68) $\sqrt{6} - \sqrt{3}$. (69) $\sqrt{6} - \sqrt{5}$.
 (70) 1.

Answers.

- (43) 5, -3. (44) 29, -70. (45) 70, -29.
 (46) 3, $-\frac{4}{5}$. (47) 1, $\frac{3}{5}$. (48) 24, $\frac{42}{5}$.
 (49) 8, -8. (50) $2 + \sqrt{3}$, $-2(2 + \sqrt{3})$.
 (51) 8, $\frac{125}{64}$. (52) 8, $(-\frac{73}{7}\sqrt{2})^2$.
 (53) ± 2 , $\pm\sqrt{10}$. (54) $\frac{17}{4}$, $\frac{1}{4}$.
 (55) 4, $\frac{1}{4}$. (56) 16, $(-\frac{71}{5})^4$.
 (57) $(-1)^{\frac{4}{3}}$, $(\frac{7}{3})^{\frac{4}{3}}$. (58) 4, -7.
 (59) 2^{π} , $\frac{1}{2^{\pi}}$. (60) 9, $-\frac{18}{5}$. (61) ± 5 .
 (62) $\frac{\pm\sqrt{4ab - b^2}}{2}$. (63) 16, 0.
 (64) 78, 3. (65) 0, $\frac{a\{1 \pm \sqrt{(-8)}\}^6}{36}$.
 (66) 0, $\frac{\pm\sqrt{3}}{2}a$. (67) $x^2 = \frac{n}{n-2}$ or $\frac{n}{n+1}$.
 (68) $x^2 = -ab \pm \frac{1}{2}\sqrt{(3a^4 + 3b^4 - 6a^2b^2)}$.
 (69) $\sqrt{x} = \frac{-(2+a) \pm \sqrt{(2a^3 + 3a^2)}}{2+2a}$.

Answers.

- (8) 2. (9) 6, $-\frac{75}{4}$. (10) 4, $-8\frac{27}{5}$.
 (11) a, b. (12) 3, $-\frac{77}{4}$. (13) 2, -3.
 (14) 27, 5. (15) -7. (16) $\frac{9 \pm \sqrt{(745)}}{2}$.
 (17) $9\frac{75}{77}$, -77. (18) $\frac{3}{2}$, $-\frac{5}{6}$. (19) 7, $\frac{4}{5}$.
 (20) 11, -73. (21) 5, -2. (22) 1, $-\frac{2}{9}$.
 (23) $\frac{3 \pm \sqrt{(687)}}{74}$. (24) 3, 2. (25) 80, -720.
 (26) 2, $\sqrt[3]{(-4)}$. (27) $\pm\sqrt{3}$, $\pm\sqrt{(-7)}$.
 (28) ± 3 , $\pm\sqrt{(-7)}$. (29) 4, $\sqrt[3]{49}$.
 (30) $\sqrt[3]{3}$, $\sqrt[3]{(-23)}$. (31) 25, 3.
 (32) 4, $-\frac{44}{3}$. (33) $4\frac{7}{4}$, $\frac{7}{4}$.
 (34) 78, 3. (35) $\frac{n}{n-a}$, $\frac{n}{9n-a}$.
 (36) ± 7 , ± 5 . (37) $\frac{3}{5}$, -7.
 (38) 7, $-\frac{7}{9}$. (39) 3, $-\frac{24}{73}$.
 (40) 2, 16. (41) -2, -7b. (42) 3, -5.

Answers.

(95) $1, \frac{47-44\sqrt{6}}{23}$. (96) $1, \frac{(\sqrt{a}+\sqrt{b})^2+4}{(\sqrt{a}-\sqrt{b})^2+4}$

(97) $x = \frac{5}{4}$. (98) $0, -1$.

(99) $0, \frac{1}{2}\{a+b+c \pm \sqrt{(a^2+b^2+c^2-2bc-2ac-2ab)}\}$.

(100) $0, \pm\sqrt{(a^2+b^2)}$.

General Properties of Equations.

(1) $3(x-5)(x+\frac{5}{3})$. (2) $(x+60)(x+13)$.

(3) $2(x+2)(x-\frac{3}{2})$. (4) $(x+62)(x+26)$.

(5) $5(x-5)(x+\frac{22}{5})$. (6) $x^2+14x+48=0$.

(7) $x^2-9x+20=0$. (8) $x^2+x-2=0$.

(9) $x^2-2x-4=0$. (10) $x^2+x=6$.

(11) $x^2-\frac{73}{3}x=\frac{10}{3}$. (12) $x^2-(a+b)x=ab$.

(13) $x^2-\frac{58}{21}x=-1$. (14) $x^2+10x=-21$.

Answers.

(70) $1, \frac{c^2-2}{(c+2)^2}$. (71) $0, \pm \frac{a\sqrt{3}}{2}$.

(72) $2a, -2a$. (73) $1, -\frac{25}{3}$.

(74) $1, \frac{7}{21}$. (75) $\pm 2a, \pm 2a\sqrt{-7}$.

(76) $x^n=0, \text{ or } \frac{4c^2a}{(c^2-1)^2b}$. (77) $\frac{7}{2}, -\frac{25}{6}$.

(78) $\pm a, \pm \frac{1}{a}$. (79) $\pm \frac{5a}{3}, \pm \frac{a\sqrt{-34}}{3}$.

(80) $5, -8$. (81) $\frac{a}{2}(1 \pm \sqrt{5})$. (82) $\pm\sqrt{2}$.

(83) $x^2 = \frac{m^4-4m^2}{4(m^2-7)}$. (84) $x^2=9$.

(85) $x^2 = \frac{a^4-b^4}{7a^2-2b^2}$. (86) $x^2 = \frac{2 \pm \sqrt{2}}{2}$.

(87) $\{c \pm \sqrt{(c^2-7)}\}^{\frac{2pq}{q-p}}$. (88) $0, \frac{16}{25}$.

(89) $\pm 2a, \pm a\sqrt{-6}$. (90) $\frac{3}{2}, \frac{2}{3}$.

(91) 5 . (92) $0, \frac{4(a+b)(a^2+b^2)}{3a^2+3b^2+10ab}$.

(93) $8, -\frac{23}{5}$. (94) $\frac{ac^2}{b^2}$.

Answers.

- (20) $x=3, y=7.$ (21) $x=5, y=4.$
 (22) $x = \frac{b}{2a}(a^2+1), y = \frac{b}{2a}(a^2-1).$
 (23) $x=10, y=75.$ (24) $x=9, y=3.$
 (25) $x=4, y=2.$
 (26) $x=2, 5; y=6, 3.$
 (27) $x = \pm 7, \pm 4; y = \pm 4, \pm 7.$
 (28) $x = -1, \frac{5}{3}; y = -1, \frac{2}{3}.$
 (29) $x=1, y=1.$ (30) $x = \pm 3, \pm 8; y = \dots$
 (31) $x=5, \frac{333}{28}; y=9, \frac{370}{84}.$
 (32) $x = \pm 3 \pm 36; y = \pm 5, \mp \frac{23}{2}.$
 (33) $x = \pm 3, \pm \frac{5}{\sqrt{2}}; y = \pm 2, \pm \frac{1}{\sqrt{2}}.$
 (34) $x = \pm 2, \pm \sqrt{\frac{2}{5}}; y = \pm \frac{1}{2}, \mp 2\sqrt{\frac{2}{5}}.$
 (35) $x = \pm 3, \pm \frac{8}{\sqrt{6}}; y = \pm 7, \pm \frac{1}{\sqrt{6}}.$
 (36) $x = \pm 4, \pm 3\sqrt{3}; y = \pm 5 \pm \sqrt{3}.$

Answers.

(15) $x^2 - \frac{7}{4}x = \frac{15}{8}.$

Equations involving several unknown quantities.

- (1) $x=2, 12\frac{5}{13}; y=3, -\frac{6}{13}.$
 (2) $x=17, 9; y=9, 17.$
 (3) $x=75, -13; y=13, -75.$
 (4) $x=7, 4; y=4, 7.$
 (5) $x=5, 7; y=7, 5.$
 (6) $x=5, y=3.$ (7) $x=2, y=3.$
 (8) $x=6, y=5.$ (9) $x=3, y=4.$
 (10) $x=5, y=2.$ (11) $x=4, y=3.$
 (12) $x=7, y=4.$ (13) $x=25, y=16.$
 (14) $x=5, y=4.$ (15) $x=4, y=2.$
 (16) $x=4, y=5.$ (17) $x=3, y=2.$
 (18) $x=3, y=2.$ (19) $x=6, y=5.$

Answers.

- (52) $x = 3, 2; y = 2, 3.$
 (53) $x = 8, 4; y = 4, 8.$
 (54) $x = 5, 13; y = 4, 12.$
 (55) $x = 4, y = 1.$ (56) $x = 7, 4; y = 4, 1.$
 (57) $x = 2, 3; y = 3, 2.$
 (58) $x = \pm 2, y = \pm 2; \text{ or } x = \pm 2, y = \mp 2.$
 (59) $x = 3, y = 1; x = 1, y = 3.$
 (60) $x = 5, -2; y = 2, -5.$
 (61) $x = \pm 2, \pm 7; y = \pm 7, \pm 2.$
 (62) $x = \frac{7}{4}(9 \pm \sqrt{73}), y = \frac{7}{4}(9 \mp \sqrt{73}).$
 (63) $x = \pm 3, \pm 2; y = \pm 2, \pm 3.$
 (64) $x = \pm 5 \pm 3; y = \pm 3, \pm 5.$
 (65) $x = \pm 3, \pm 2; y = \pm 2, \pm 3.$
 (66) $x = \pm \sqrt{-3}, \pm \sqrt{3}; y = 3 \mp \sqrt{-3}, \pm 2\sqrt{3}$
 $\text{or } \mp 2\sqrt{3}.$
 (67) $x = 8, 2; y = 2, 8.$

Answers.

- (37) $x = \pm \frac{75}{\sqrt{27}}, y = \pm \frac{3}{\sqrt{27}}.$
 (38) $x = 3, -\frac{53}{27}; y = -4, \frac{227}{27}.$
 (39) $x = \pm \sqrt{\frac{5}{2}}; y = 2 \mp \sqrt{\frac{5}{2}}.$
 (40) $x = \pm 6, y = \pm 3, \mp 3.$
 (41) $x = \pm 3\sqrt{2}; y = \pm \sqrt{2}, \mp \sqrt{2}.$
 (42) $x = 0, 1; y = 0, -\frac{12}{5}.$
 (43) $x = 0, 4; y = 0, 5.$
 (44) $x = 0, 15; y = 0, 45.$
 (45) $x = 0, 2, \pm \sqrt{2}; y = 0, 2, 2 \mp \sqrt{2}.$
 (46) $x = 0, 4, 2; y = 0, 2, -4.$
 (47) $x = 5, \frac{27}{5}; y = 3, \frac{7}{5}.$
 (48) $x = 4, 2; y = 2, 4.$
 (49) $x = 2, 0; y = 0, -2.$
 (50) $x = 7, 4; y = 4, 1.$
 (51) $x = 7, 10; y = 10, 7.$

Answers.

(68) $x = 9, 4; y = 4, 9.$

(69) $x = 8, 64; y = 64, 8.$

(70) $x = 4, 9; y = 9, 4.$

(71) $x = 2, 8; y = 8, 2.$

(72) $x = \pm 7, y = 3.$ (73) $x = \frac{a}{2}, y = \frac{b}{2}.$

(74) $x^2 = \pm \frac{5a^2}{3}, \pm a^2; y^2 = \frac{4a^2}{3}, 0.$

(75) $x = 5, y = 3.$

(76) $x = 0, 2a; y = b, -b; z = 0, -c.$

(77) $x = \frac{7}{2}, \frac{5}{26}; y = \frac{7}{3}, \frac{75}{73}; z = \frac{7}{4}, \frac{75}{44}.$

(78) $x = \pm 3, y = \pm \frac{7}{2}, z = \pm \frac{3}{2}.$

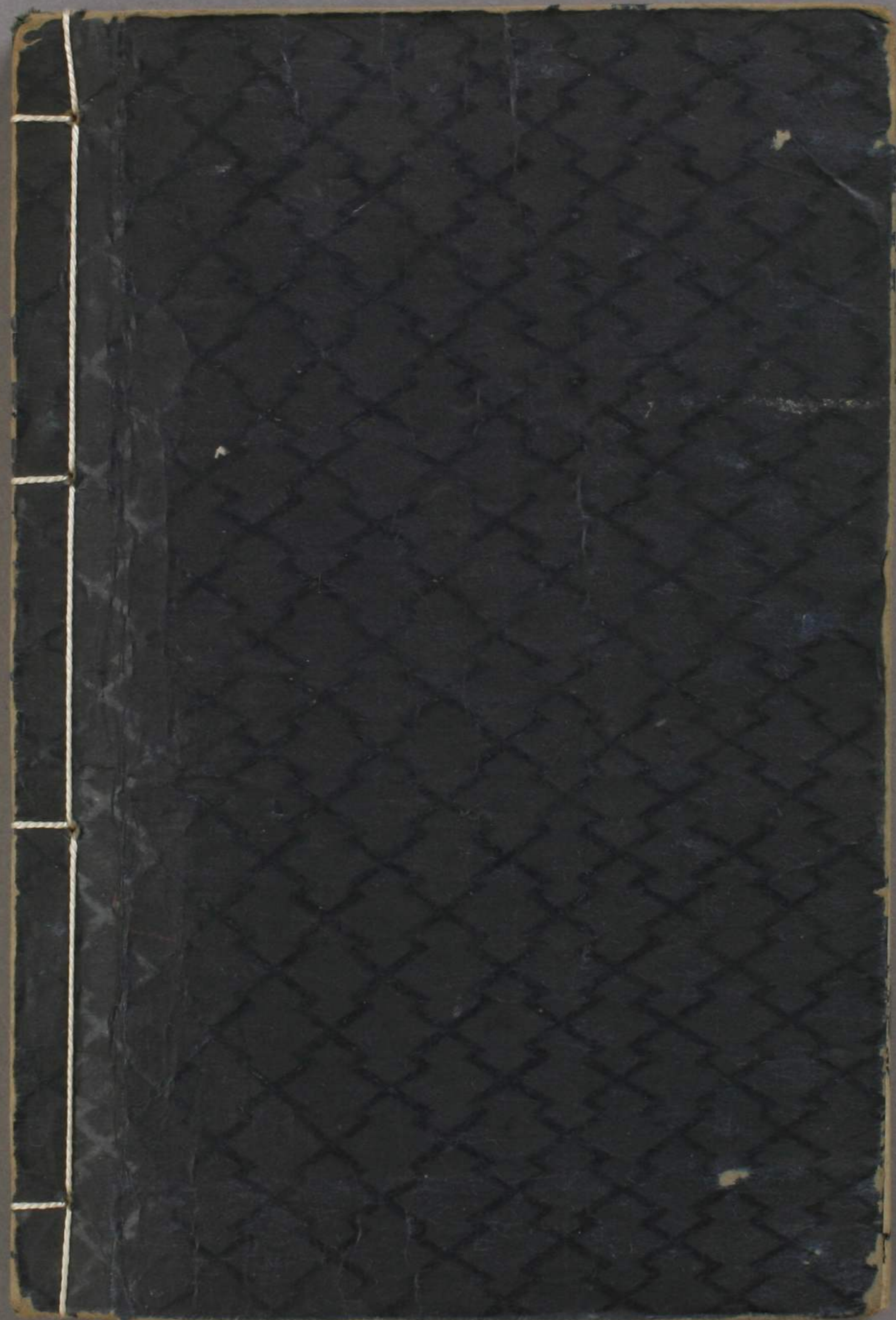
(79) $z = 6, x = 4, y = 5;$

$z = -\frac{5}{2}, x = \frac{355}{42}, y = \frac{790}{21}.$

(80) $x = 2, 2, 3, 3, 4, 4.$

$y = 3, 4, 2, 4, 2, 3.$

$z = 4, 3, 4, 2, 3, 2.$



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