幾何問題解式

荒川重平
中川將行　同著

…題解式

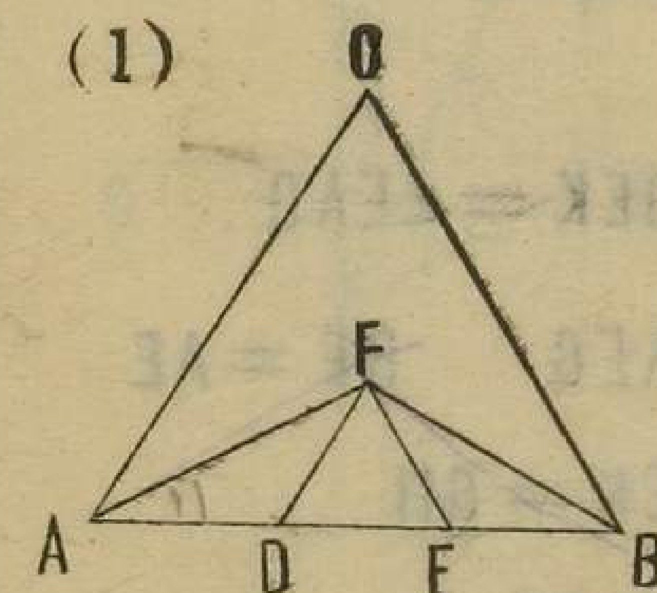
書肆
積玉圃
種玉堂 藏梓

幾何問題解

卷之一

中川將行　荒川重平
同著

(1)



ABヲ直線トス

(画法)

ABC ＝ 等邊三角形

$\angle BAF = \frac{\angle A}{2}$,

$\angle ABF = \frac{\angle B}{2}$,　FD // CA,　FE // CB,

D, E ハ分点ナリ

荒川重平
中川將行 同著

幾何問題解式

積玉圃
種玉堂 藏梓

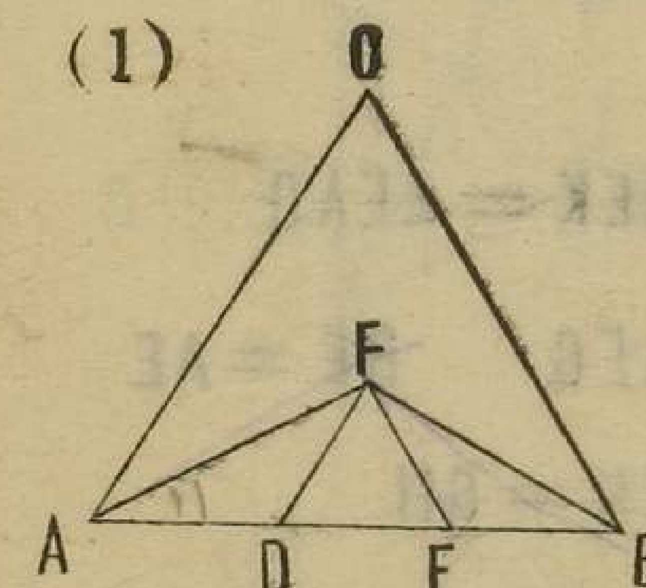
書肆

幾何問題解

巻之一

中川將行　荒川重平
同著

(1)



AB ヲ直線トス

(画法)

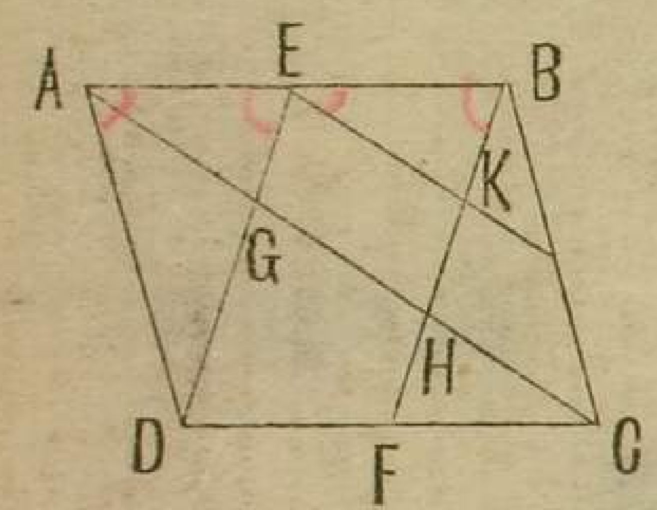
ABC ＝ 等邊三角形

$\angle BAF = \frac{\angle A}{2}$,

$\angle ABF = \frac{\angle B}{2}$,　FD // CA,　FE // CB,

D, E ハ分点ナリ

(2)



ABCDヲ平行形、E, FヲABCDノ折半点、EK // ACトス

(証)

$EB = \frac{AB}{2} = \frac{DC}{2} = DF$

∵ EB ⫲ DF　　∴ ED ⫲ BF

∵ AC // EK　　∴ ∠BEK = ∠EAG

同理ニテ ∠EBK = ∠AEG　　BE = AE

∴ AG = EK = GH

同法ニテ CH = GH

∴ AG = GH = HC

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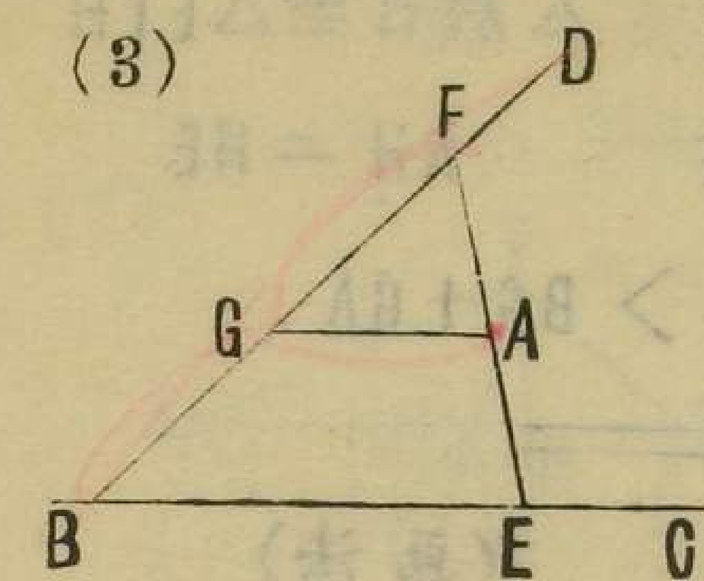
(3)



BC, BDヲ二直線、Aヲ固有点トス

(画法)

AG // CB、GF = GB、FAEハ求ムルモノナリ

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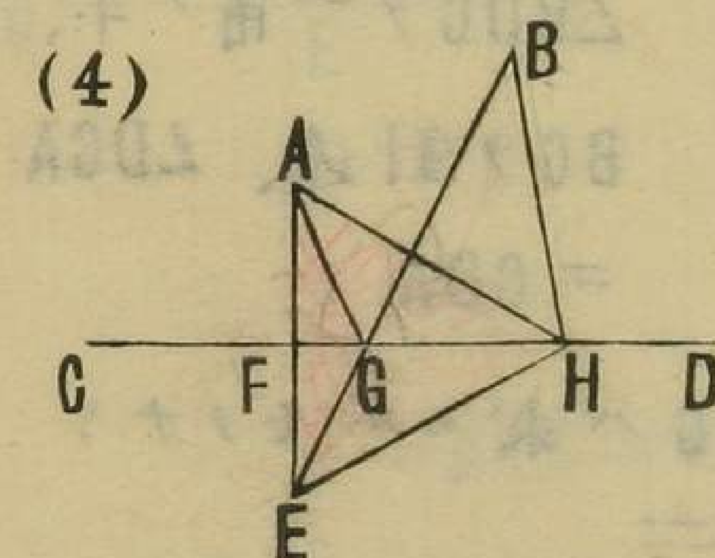
(4)



(1) A, Bヲ二点 CDヲ直線トス

(画法)

AFE ⊥ CD、

FE = FA

AG, BGハ求ムル者ナリ

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(2) CD中他ノ点Hヲ採リ

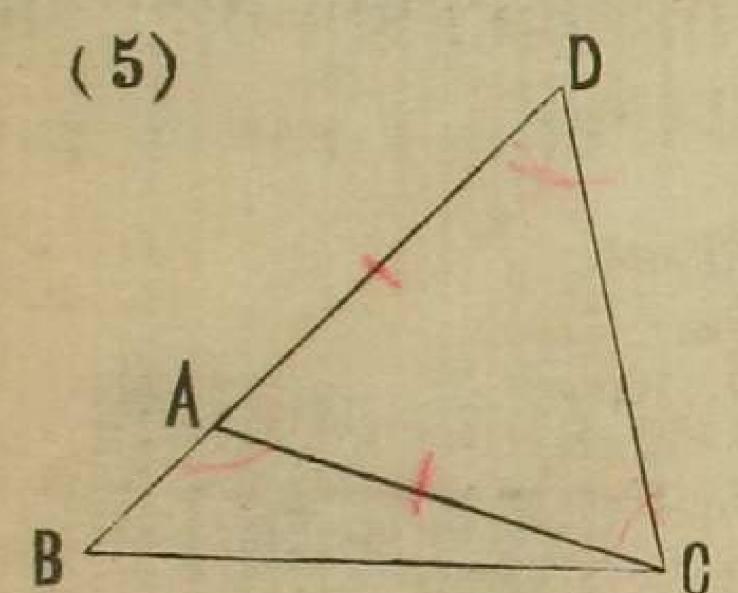
(証) ∵ BH + HE ＞ BE

但シ△AFG≌△EFG　△AFH≌△EFH

∴ AG＝GE　AH＝HE

∴ BH＋HA＞BG＋GA

(5)



(画法)

BDヲ二邊ノ和、

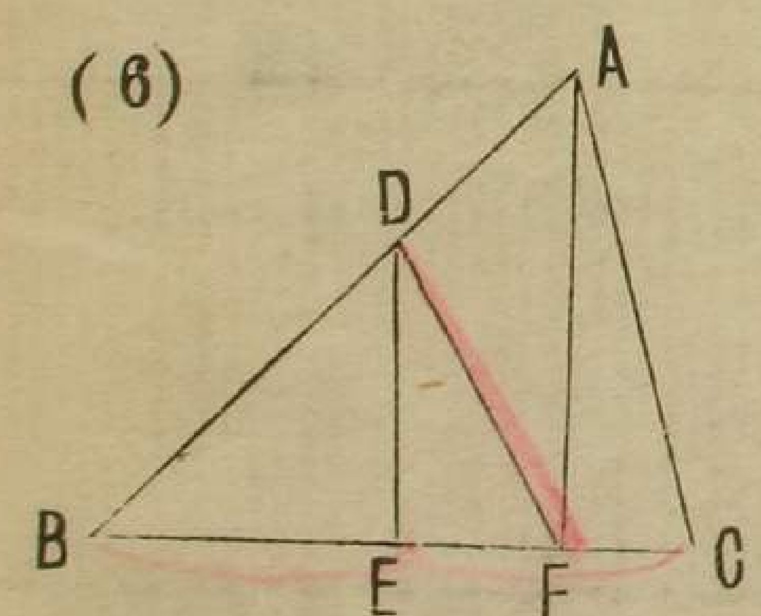
∠BDCヲ一角ノ半、

BCヲ對邊、∠DCA

＝CDA

ABCハ求ムルモノナリ

(6)



ABCヲ三角形、

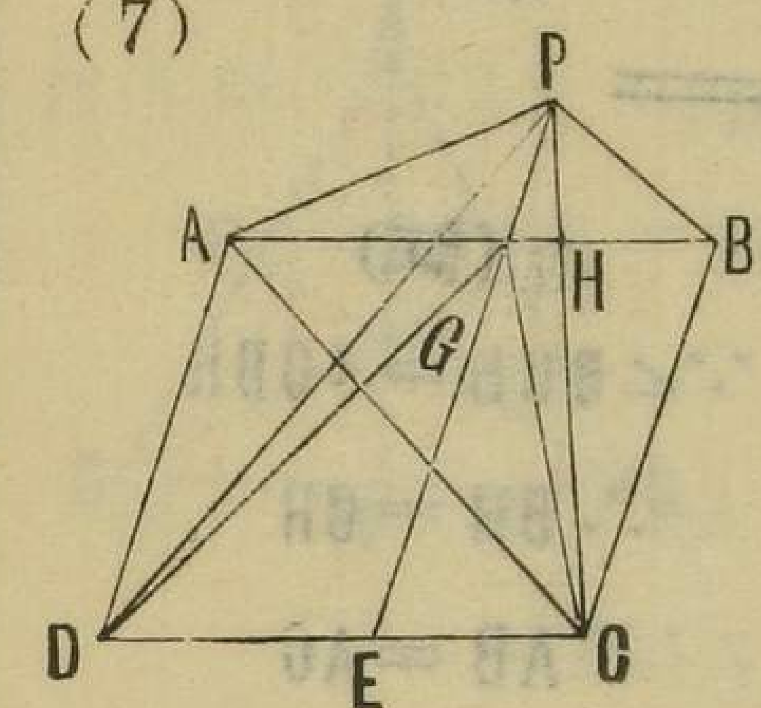
DヲBA中ノ一点トス

(画法)

BE＝EC、AF∥DE

DFハ求ムルモノ也

(7)



ABCDヲ平行形、

Pヲ本形外ノ一

点、PGE∥BCトス

(証)

∵ PGE∥BC

∴ △CBP＝△CBG

∴ △PHB＝△CHG

△DAG＝△AGC＝△DAP

∴ △PHB＋△DAP＝△AGC＋△CHG＝△ACH

∴ △APH＋△PHB＋△DAP＝△APH＋△ACH

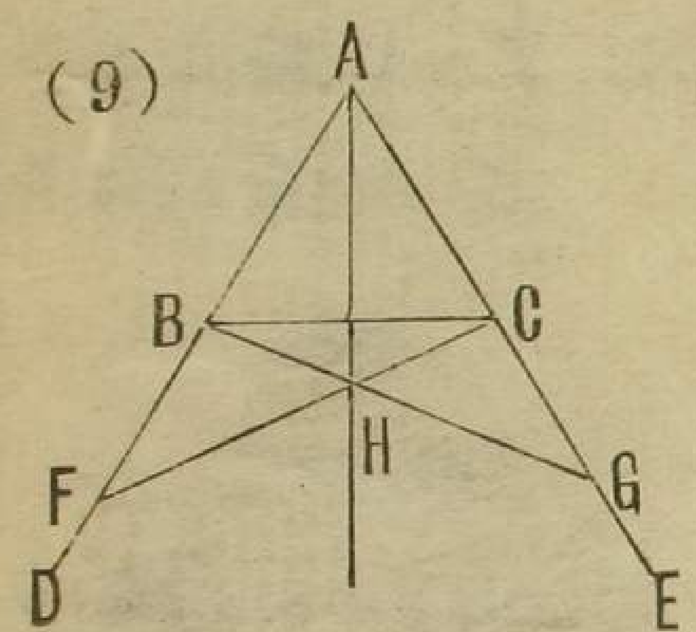
∴ △APB＋△DAP＝△APC

P点内ニアルトキハ

△APB∽△DAP＝△APC

(8) 画法ヲ説カザルモ明カナルベシ

(9)



(証)

$\because \angle BCH = \angle CBH$

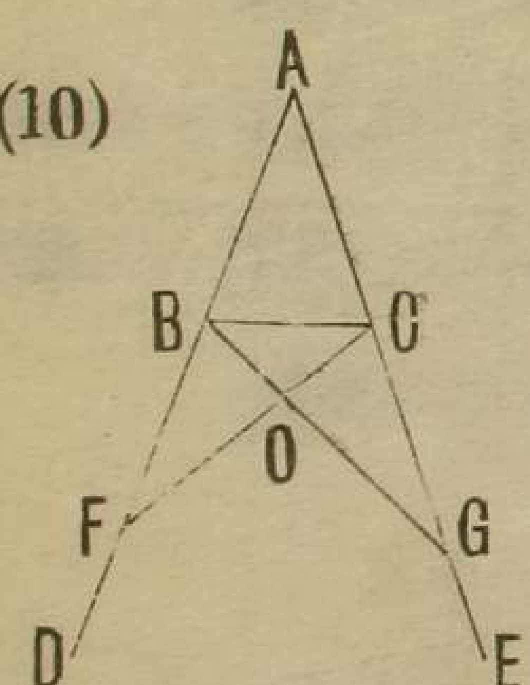
$\therefore BH = CH$

$AB = AC$

$AH = AH$

$\therefore \angle BAH = \angle CAH$

(10)



(証)

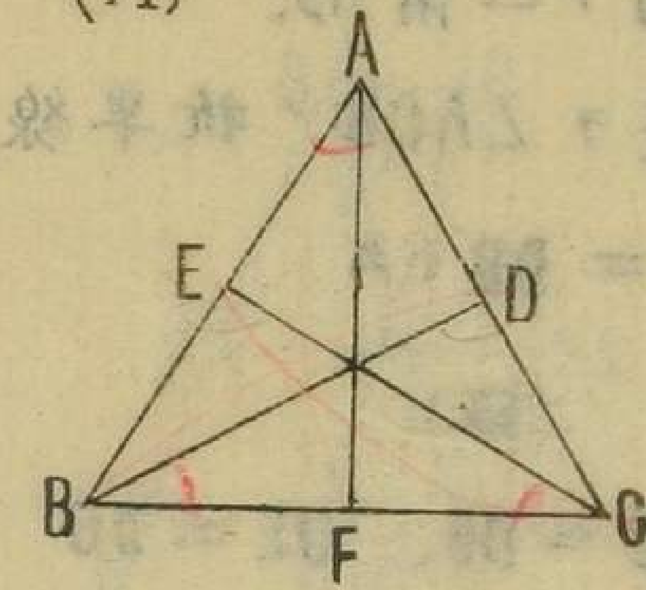
$\because \angle ABC = \angle FBG = \angle ACB$

$\therefore \angle ABC + \angle FBC = \angle ABC + \angle ACB$

$\therefore \angle A = \angle OBC = \angle OCB$

$\therefore \angle BOF = \angle OBC + \angle OCB = 2\angle A$

(11)



ABCヲ二等邊三角形

CE, BDヲAB, ACノ垂線、AF⊥BCトス

(証)

$\because \triangle ABF \cong \triangle ACF$

$\therefore \angle BAF = \frac{\angle A}{2}$

$\because \frac{\angle A}{2} + \angle ABC = \angle ECB + \angle ABC = r.a.$

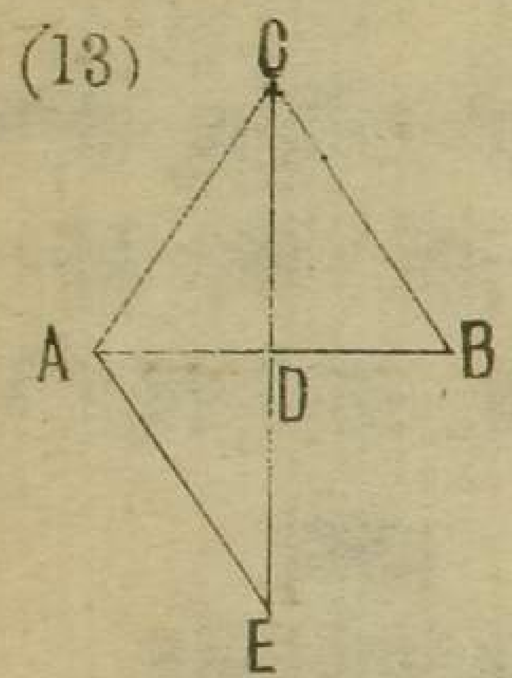
$\therefore \angle ECB = \frac{\angle A}{2} = \angle DBC$

(証)

(12) 二邊挾角相同シ

故ニ両△相同シ

(13)



ABC ヲ三角形、

CDE ヲ ∠ACB ノ折半線、

DE ＝ DC トス

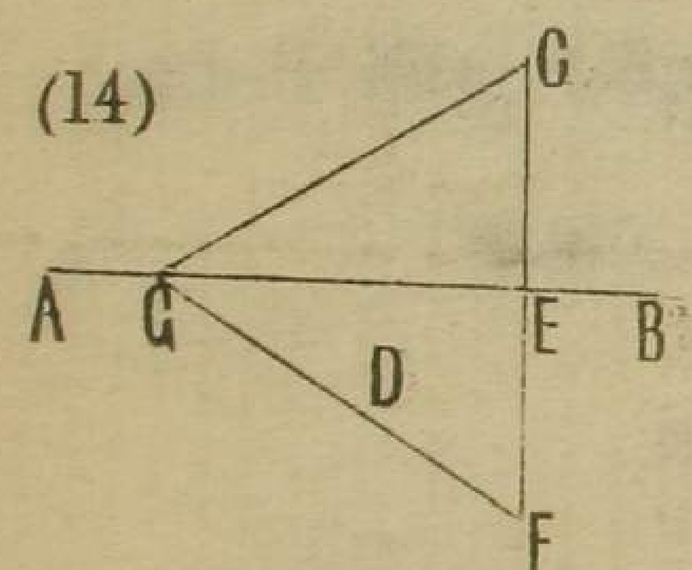
（証）

∵ AD＝DB、DE＝DC

∠ADE＝∠BDC

∴ AE＝BC、∠AED＝∠BCD＝ACD、∴ AC＝AE＝BC

(14)



AB ヲ直線、C

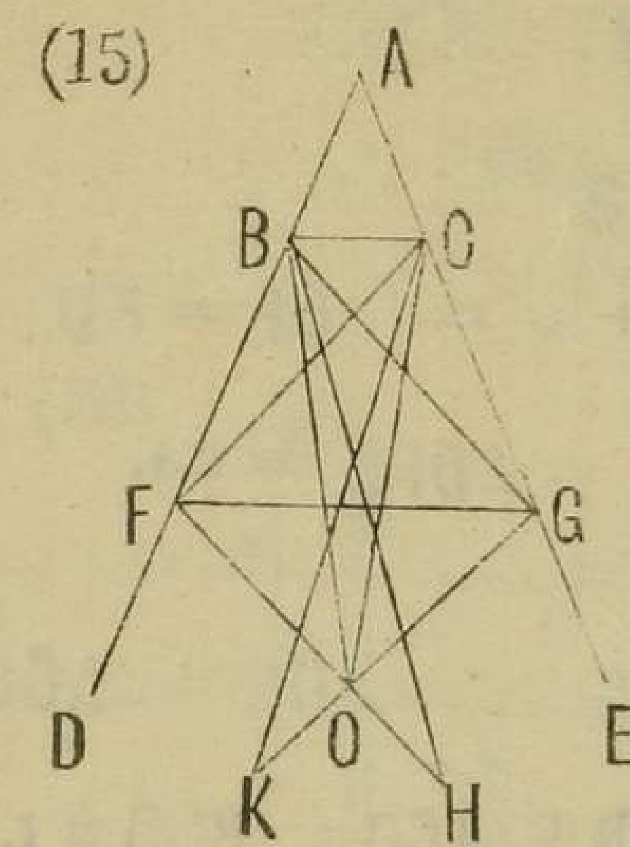
D ヲ二点トス

（画法）

CEF⊥AB、EF＝EC

FDG, GC ハ求ムルモノ也

(15)



（証）

∵ BF＝CG

FH＝GK

∠BFH＝∠CGK

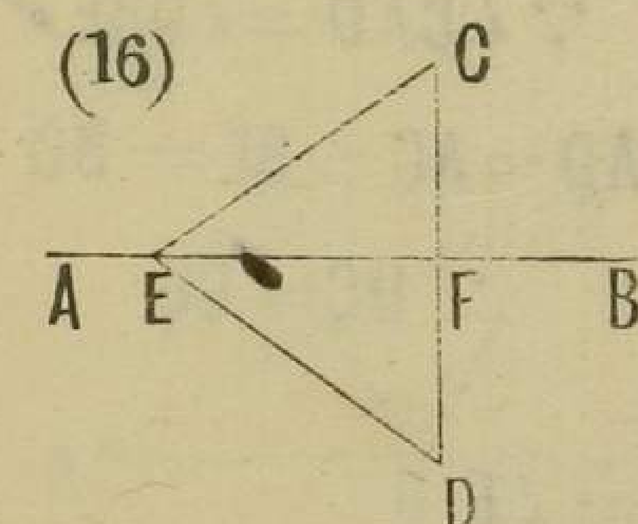
∴ BH＝CK

∵ ∠FGO＝∠GFO

∴ FO＝GO　BF＝CG

∠BFO＝∠CGO　∴ BO＝CO

(16)



AB ヲ直線、C,

D ヲ二点、

E ヲ AB 中随意点

トス

(証)

$\because CE=ED$

Fモ亦AB中ノ一点ナレハ　$CF=FD$、

$EF=EF$、　$\therefore \angle CFE=\angle DFE=r.a.$

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(17) (証)　$\because AD=AC$　$\therefore \angle ADC=\angle ACD$、

$\angle ABC+\angle ADC=\angle ACB+\angle ACD=\angle BCD=r.a.$

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(18) (証)　$\because \angle ADE=r.a.-\angle B=r.a.-\angle C=\angle E$

$\therefore AE=AD$

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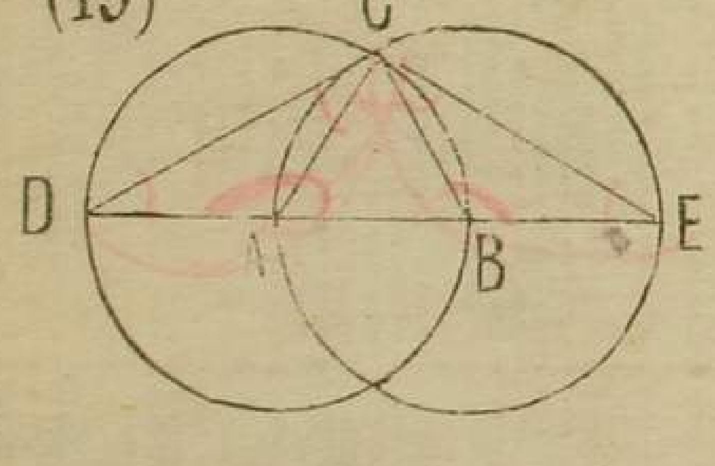
(19)



(証)

$\because \angle CAD=\angle CBE$

$AD=AC=BE=BC$

$\therefore DC=CE$

$\because \angle D=\angle ACD=\angle BCE=\angle E$

$\angle ACB=\angle CAB=\angle D+\angle ACD$

$\therefore \angle DCE=4\angle D$

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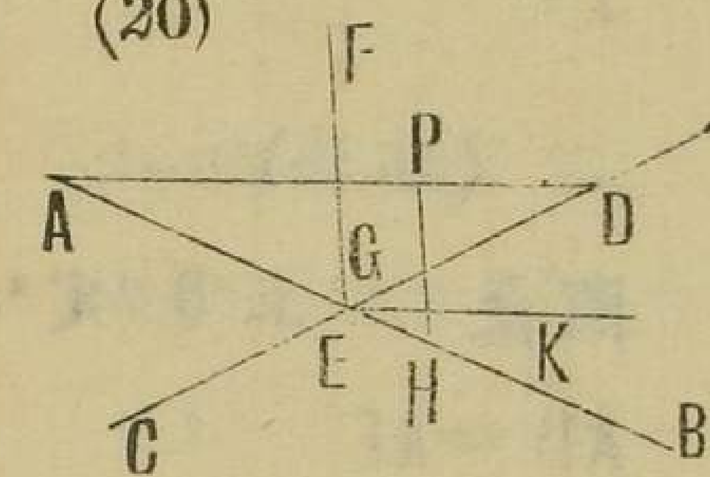
(20)



AB, CDヲ両直線、Eヲ交点、Pヲ固有点トス

(画法)

$\angle AEF=\angle DEF$　$PGH \parallel FE$

$\angle BEK=\angle DEK$　$APD \parallel EK$

PGH、APDハ求ムルモノ也

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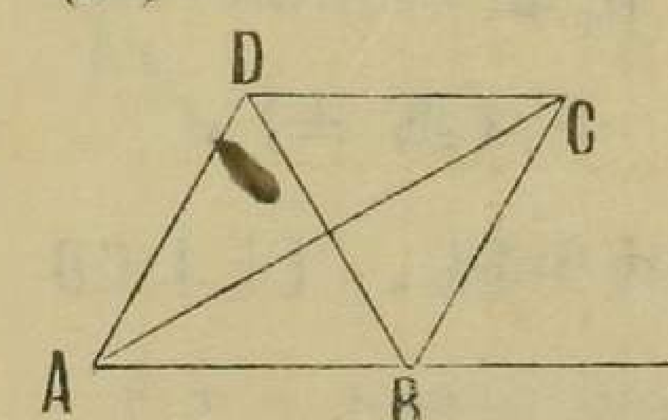
(21)



AB, ACヲ甲乙両直線、Dヲ一点トス

(画法)

$DC \parallel AB$、$CB \parallel DA$、

DBハ求ムルモノナリ

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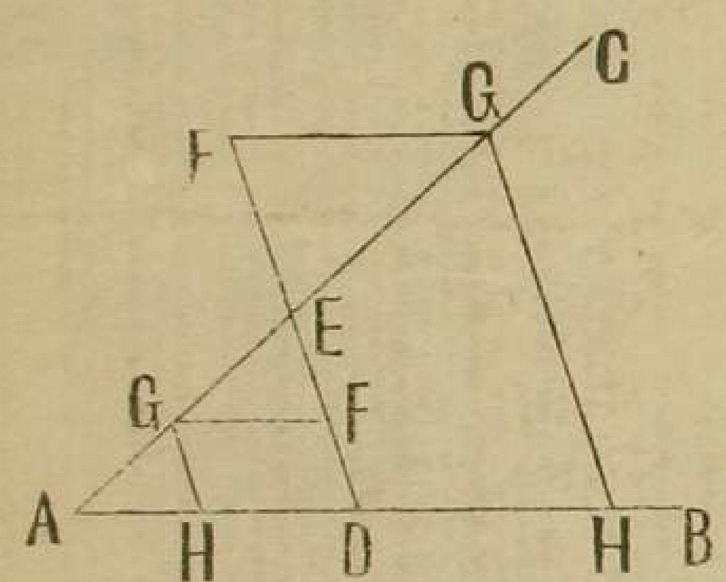
(22)



AB, AC ヲ兩直線トス

(画法)

随意ノ一点Dヲ定メ、

AD＝AE

DF＝已定線ノ長

FG∥AB、GH∥ED、GHハ求ムルモノ也

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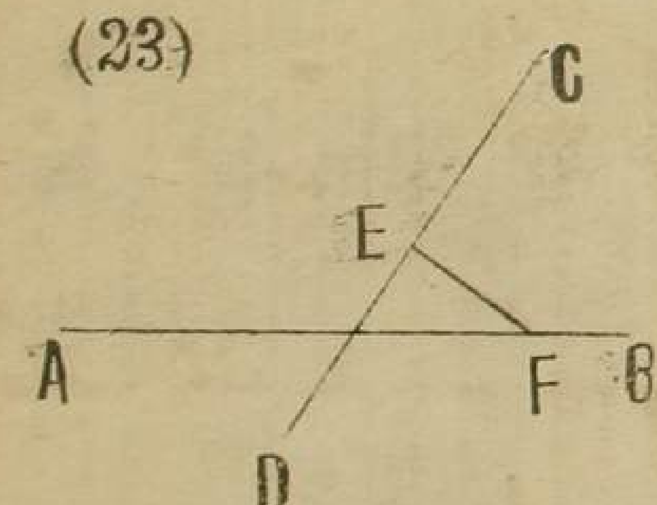
(23)



AB ヲ直線

C, D ヲ二点トス

(画法)

DE＝EC、EF⊥CD

Fハ求ムルモノナリ

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(24) (画法) 甲乙ニ直線ノ交角ヲ折半スル線丙線ニ會スル点ハ即チ求ムルモノナリ

丙線甲乙ノ間ニアリテ相平行シ且其中央ニアラザルキハ能ハサルノ場合ナリトス

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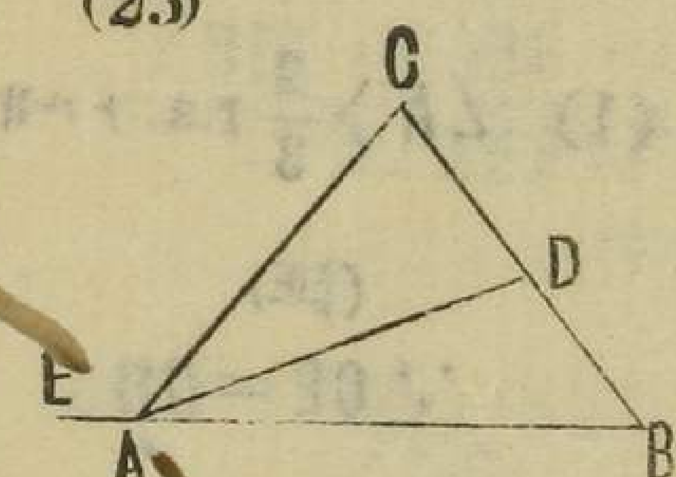
(25)



ABC ヲ二等邊三角形、AD＝AC トス、

(1) 三角共ニ鋭角ナルキ

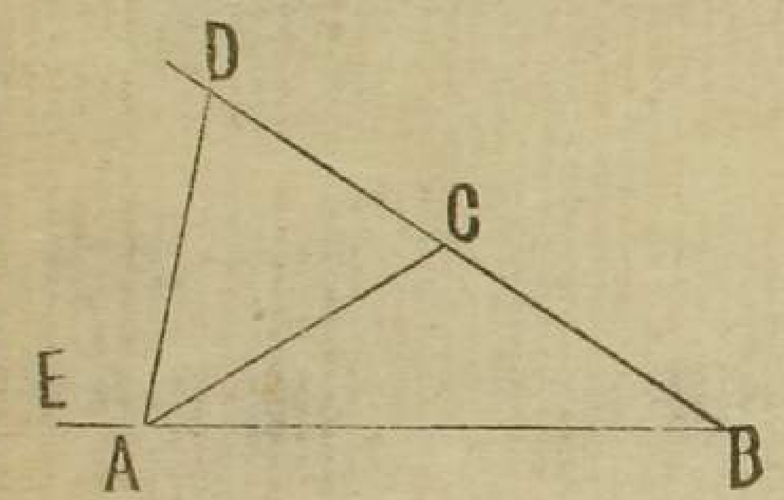
(証)

$\because \angle EAD = \angle EAC + \angle CAD$

$\angle EAC = \angle B + \angle C = \angle B + \angle CDA$

$= \angle B + \angle B + \angle BAD$

$\therefore \angle EAD = 2\angle B + \angle BAD + \angle CAD = 3\angle B$



(2) C 鈍角ナルキ

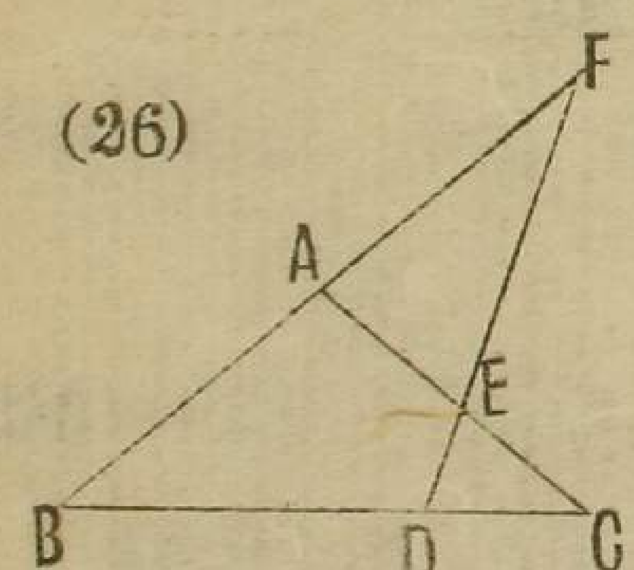
(証)

$\because \angle EAD = \angle D + \angle B$

$\angle D = \angle ACD = \angle B + \angle CAB = 2\angle B$

$\therefore \angle EAD = 3\angle B$

(26)



(1) $\angle A > \frac{2}{3}$ r.a. ナルキ

(証)

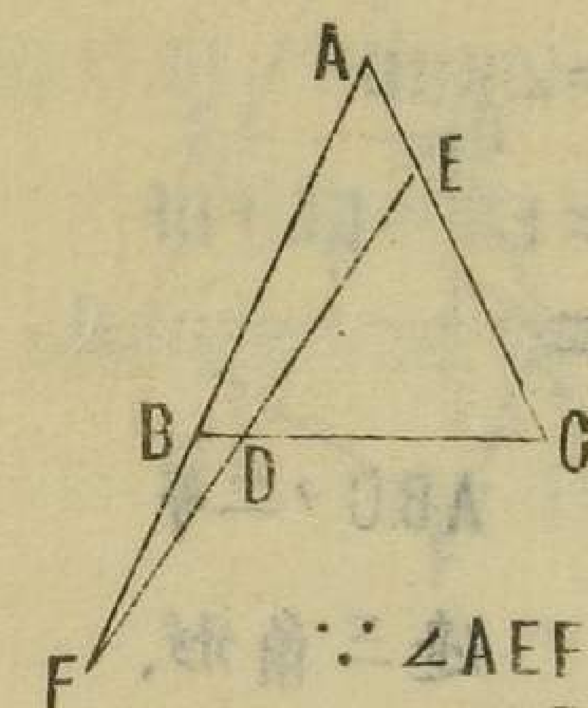
$\because CE = CD$

$\therefore \angle E = \angle CDE$

$\because \angle E + \angle EDC = \angle B + \angle BAC$

$\angle EDC = \angle B + \angle F$

$\therefore 3\angle E = 2\angle B + \angle BAC + \angle F = 2$ r.a. $+ \angle F$



(2) $\angle A < \frac{2}{3}$ r.a. ナルキ

(証)

$\because CE = CD$

$\therefore \angle AEF = \angle BDE$

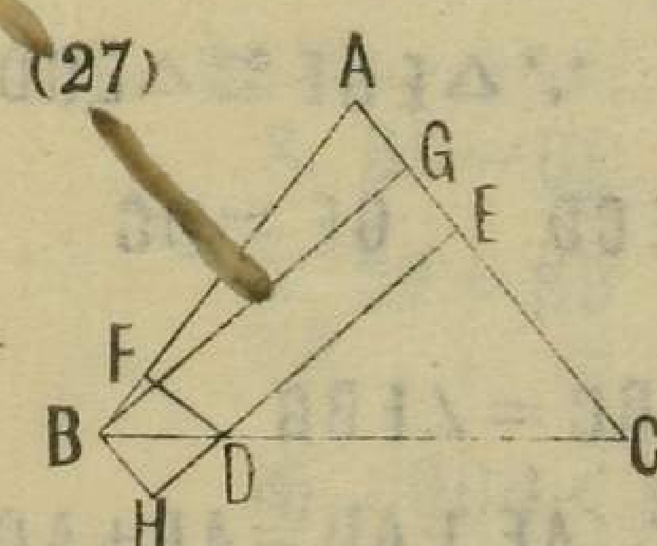
$\because \angle AEF = \angle C + \angle CDE$

$\angle BDE = \angle C + \angle CED$

$\angle BDE = \angle DBF + \angle F = \angle A + \angle C + \angle F$ (+

$\therefore 3\angle AEF = 4$ r.a. $+ \angle F$

(27)



ABCヲ二等邊三角形、

Dヲ随意ノ点、

$\angle CDE = \angle BDF = \angle CBG$

$DH = DF$ トス

(証)

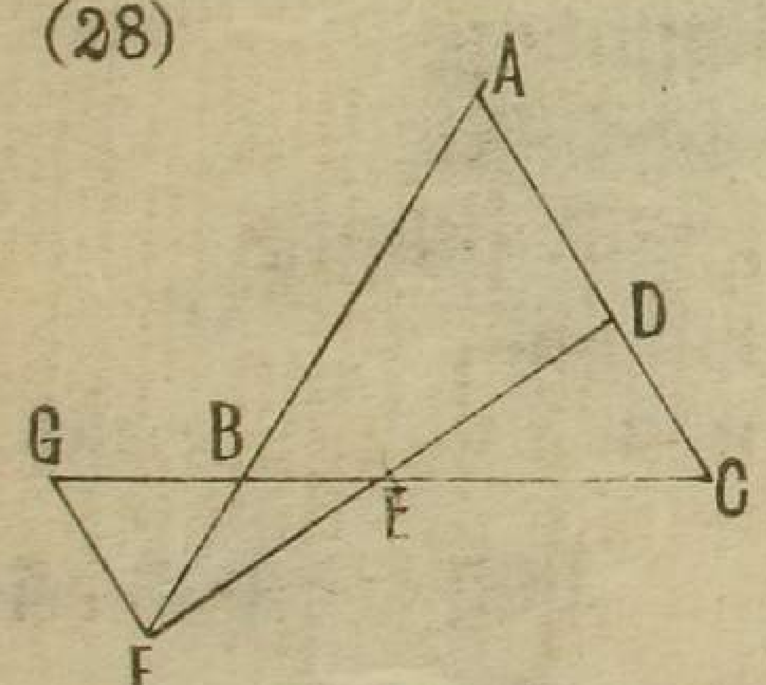
$\because \triangle BDH \cong \triangle BDF$　$\therefore \angle BHD = \angle BFD$

$\because \angle C = \angle DBF$　$\angle CBG = \angle BDF$

$\therefore \angle BGC = \angle BFD = \angle BHD$

$BG \parallel EH$　$\therefore BG = EH = DE + DF$

(28)



ABCヲ二等邊三角形、DE＝EF、EG＝CEトス

(証)

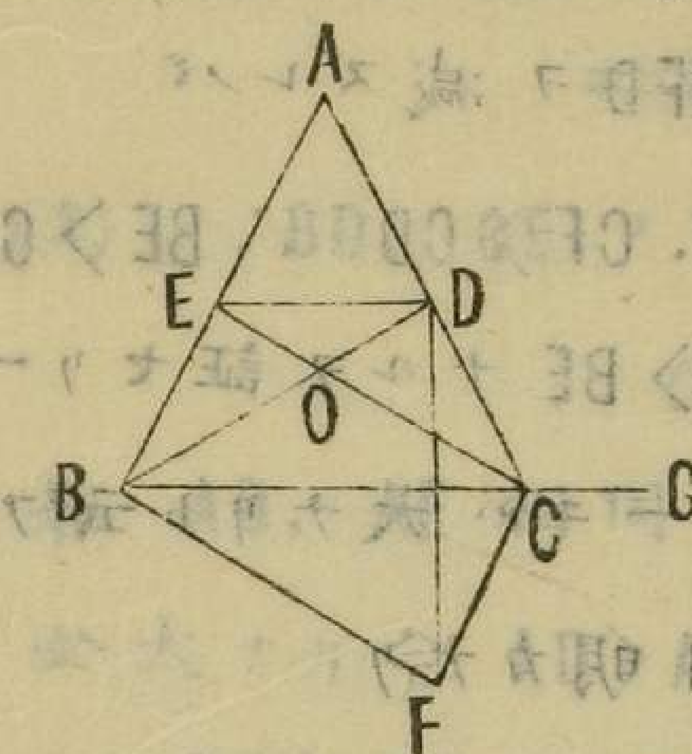
$\because \triangle EGF \cong \triangle ECD$

$\therefore \angle EGF = \angle ECD$　$GF = DC$

$\because \angle BGF = \angle ACB = \angle ABC = \angle FBG$

$\therefore BF = GF = DC$　$\therefore AF + AD = AB + AC$

(29)(30)



(1) ABCヲ三角形、BD, CEヲ底角ノ折半線トス

(証)

若$\angle OBC$、$\angle OCB$相等カラズトセバ一ハ必大ナリ今$\angle OBC > \angle OCB$ト假定ス

$\because CB = CB$, $BD = CE$　然レ氏

$\angle CBD > \angle BCE$　$\therefore CD > BE$

又 $BF = CE$　$CF = BE$ ト為ス

$\therefore BF = BD$　$\therefore \angle BFD = \angle BDF$

而 $\angle OCD < \angle OBE$ 及 $\angle COD = \angle BOE$

$\therefore \angle ODC > \angle OEB \quad \therefore \angle ODC > \angle BFC$

$\angle BDF = \angle BFD$ ヲ減ズレバ

$\angle FDC > \angle DFC \quad \therefore CF > CD \quad \therefore BE > CD$

然レトモ前巳ニ $CD > BE$ ナルヲ証セリ一

ニハ小ナリト云ヒ一ニハ大ナリト云フ

故ニ誤謬ナルコト明カナリ

然ラバ $\angle OBC > \angle OCB$ ニ非ズシテ

$\angle OBC = \angle OCB \quad \therefore \angle ABC = \angle ACB$

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(2) $\because \angle OCG = \angle BOC + \angle CBO$

$\angle OCD = \angle OBC \quad \therefore \angle ACG = \angle BOC$

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(30) (1) ABC ヲ三角形、BD, CE ヲ

AC, AB ノ垂直線トス

(証) $\because \triangle ADB \cong \triangle AEC$

$\therefore AB = AC$

---

(2) BD, CE ヲ AC, AB ノ折半線トス

(証)

若 AD, AE 相同シカラストセバ一ハ

必大ナリ今 $AD > AE$ ト假定スレバ

$\because AD > AE \quad \therefore \angle AED > \angle ADE$

$\because CE = BD, \quad ED = DE \quad CD > BE$

$\therefore \angle CED > \angle BDE$ 相加ヘテ

$\therefore \angle AEC > \angle ADB \quad \therefore \angle ODC > \angle OEB$

餘ハ (29) ノ (1) ノ (証) ト略ヽ同シ

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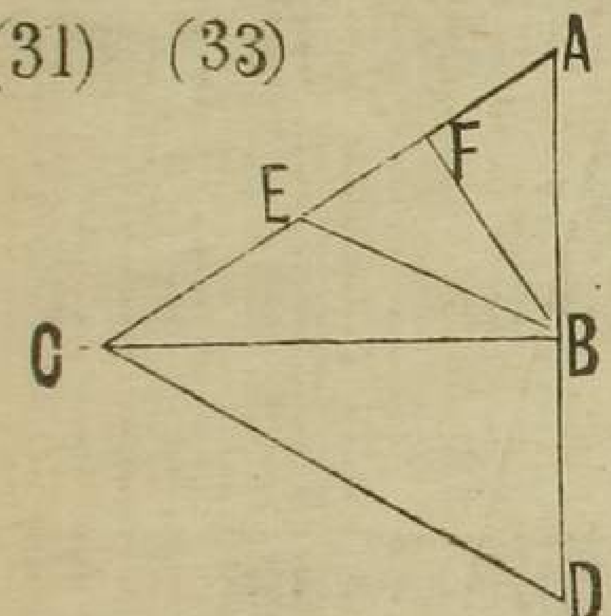
(3) BD, CE ヲ $\angle BDA = \angle CEA$ ナラシムル

線トス

(証) $\because \triangle ADB \cong \triangle AEC \quad \therefore AB = AC$

(4) DE∥BC ハ別ニ証スル迄モナシ

(31) (33)



∠BCD＝∠BCA トス

(証)

∵△ABC≌△DBC

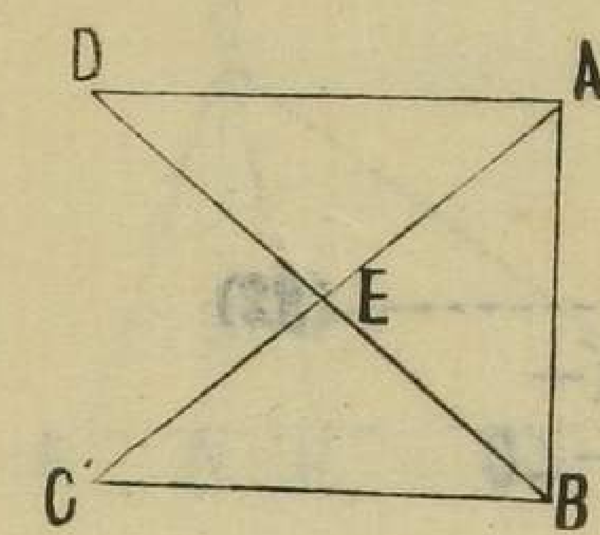
∴AB＝BD

∵∠CAD＝∠ACD＝∠ADC

∴AC＝AD＝CD＝2AB

∵CB＜AC ∴CB＜2AB

(32) ABCヲ三角形、∠ABC＝∠BAC＋∠ACB、AE＝EC ナリ



BD＝2BE トス

(証)

∵△AED≌△CEB

∴∠EAD＝∠ECB

AD＝CB

∵∠ECB＋∠CAB＝∠ABC

∴∠EAD＋∠CAB＝∠ABC

∴∠BAD＝∠ABC ∵△ABC≌△BAD

∴AC＝BD＝2BE

∵∠ACB＋∠BAC＋∠ABC＝2 r.a.

又∠ACB＋∠BAC＝∠ABC ∴∠ABC＝r.a.

∴∠ABC＝r.a. ノ片ハ AE＝EC＝BE

此証(82)ニ有用ナレバ此處ニ掲グ

(33) ABC ヲ直三角形、 BF⊥AC、 AE＝EC トス

(証) $\because \angle CBF = \angle A$

$\angle CBE = \angle C$ --------- (32)

$\therefore \angle EBF = \angle A - \angle C$

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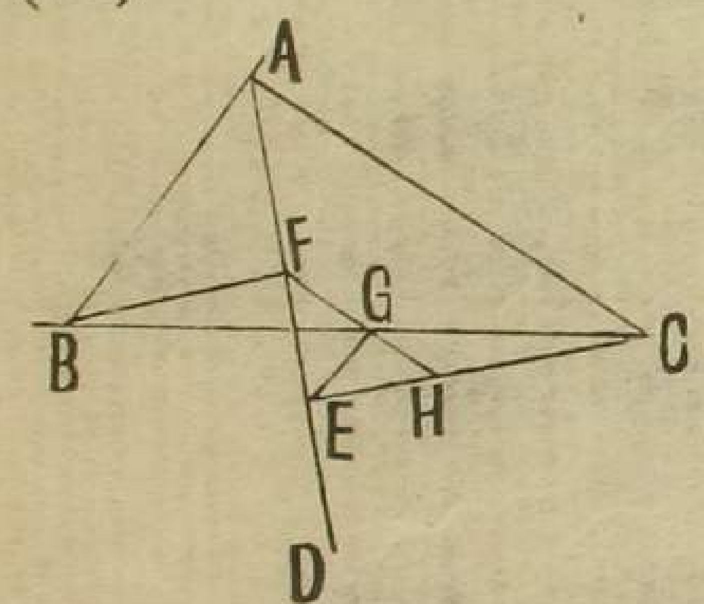
(34)



(証)

$\because BF \parallel CE$

$\therefore \angle FBG = \angle HCG$

$\angle BGF = \angle CGH$

$BG = CG$

$\therefore FG = GH$

FEH ハ直三角形ニテ G ハ弦ノ正中点ナレバ

$EG = GF$ ----- (32)

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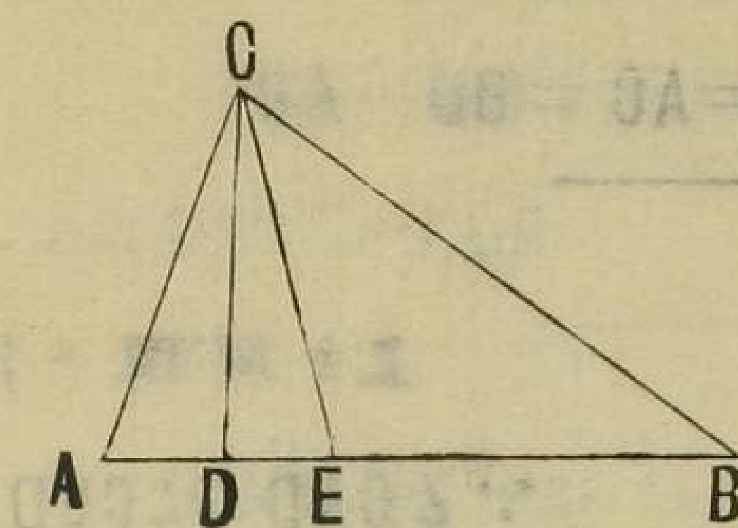
(35) (36)



(1) ABC ヲ三角形、CD ヲ垂線、CE ヲ頂角ノ折半線トス

(証)

$\because \angle A + \angle ACE = r.a. + \angle DCE$

$\angle B + \angle BCE = r.a. - \angle DCE$

$\therefore \angle A - \angle B = 2\angle DCE$

A 鈍角ナルトキモ亦相同シ

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(37) ABC ヲ三角形、 $\angle A = 2\angle B$

(1) CD ヲ垂線、 DE＝AD トス

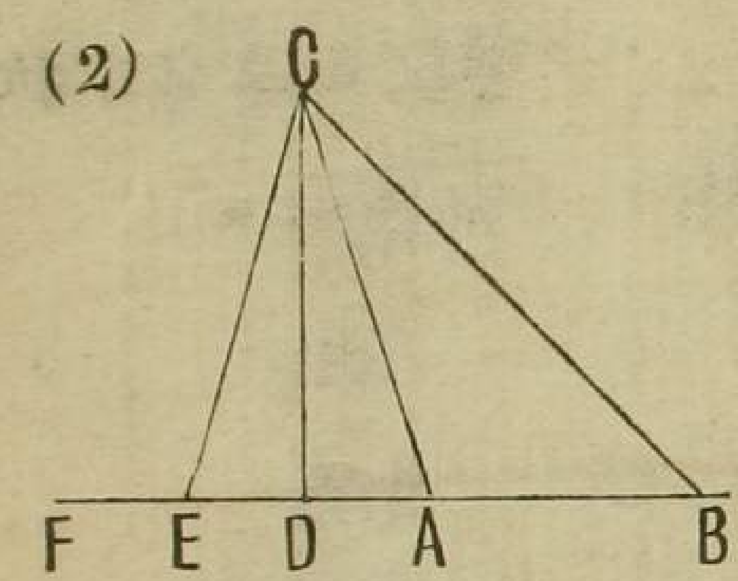
(証)

$\because \triangle ADC \cong \triangle EDC, \therefore \angle A = \angle CED = 2\angle B$

$\therefore \angle BCE = \angle B$

$\therefore BE = CE = AC = BD - AD$

(2)



上ト同理ニテ

$\because \angle CAD = \angle CED$

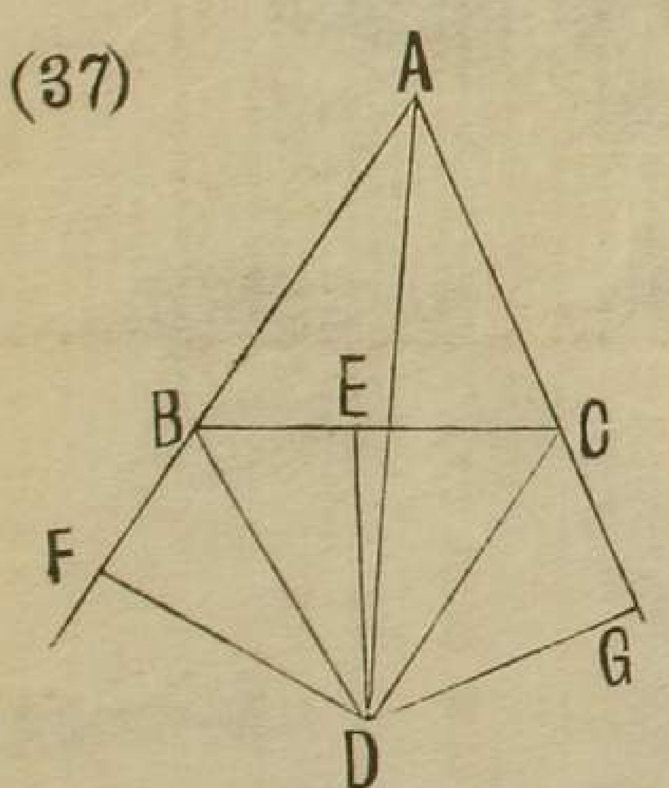
$\therefore \angle CAB = \angle CEF$

$= 2\angle B$

$\because \angle BCE = \angle B$

$\therefore BE = CE = CA = BD + AD$

(37)



ABCヲ三角形、BD, CDヲ外角ノ折半線、DE, DF, DG $\perp$ BC, AF, AGトス

(証)

$\because \triangle BFD \cong \triangle BED$

$\therefore DF = DE$

$\because \triangle CED \cong \triangle CGD$ $\therefore DE = DG$

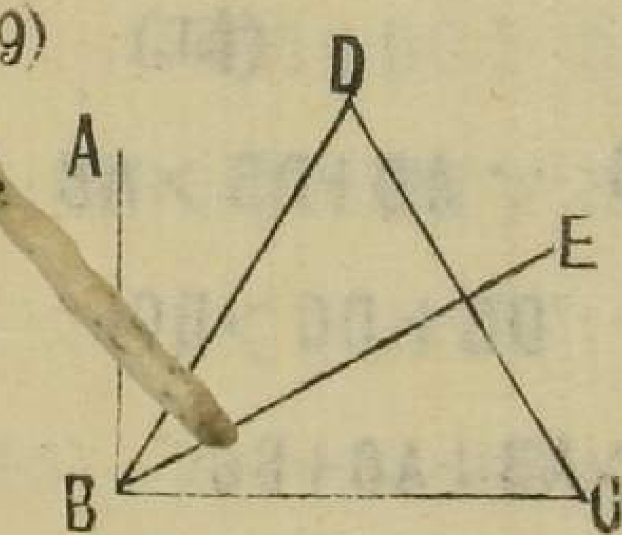
$\because AD^2 = DF^2 + AF^2 = DG^2 + AG^2$

$DF = DG$、 $\therefore AF = AG$

$\because \triangle AFD \cong \triangle AGD$ $\therefore \angle DAF = \angle DAG$

(38) 求ムル線ハ大小邊ノ和半ニ同シ

(39)



ABCヲ直角トス

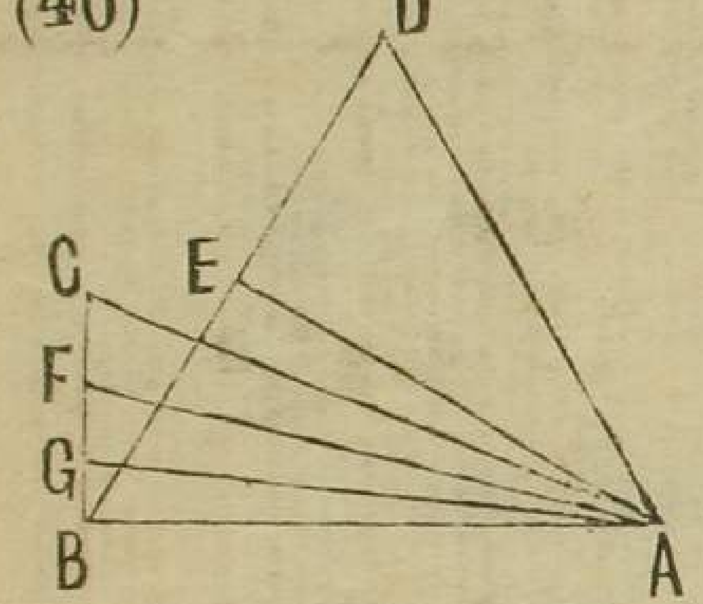
(画法)

$BC = BD = DC$

$\angle CBE = \angle DBE$

BD, BEハ分線ナリ

(40)



ABC ヲ直三角形、
$\angle C = 3\angle A$ トス

(画法)

$BD = BA = AD$

$\angle BAE = \angle DAE$

$\angle CAF = \angle CAE =$

$\angle FAG$, AF、AG ハ分線ナリ

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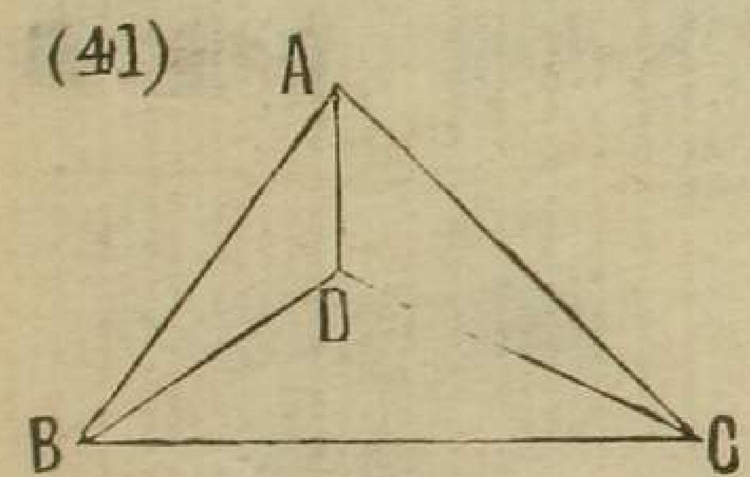
(41)



ABC ヲ三角形、
D ヲ其内ノ一点トス

(証)

$\because AD + DB > AB$

$AD + DC > AC \qquad DB + DC > BC$

$\therefore 2(AD + DB + DC) > AB + AC + BC$

$\therefore AD + DB + DC > \frac{1}{2}(AB + AC + BC)$

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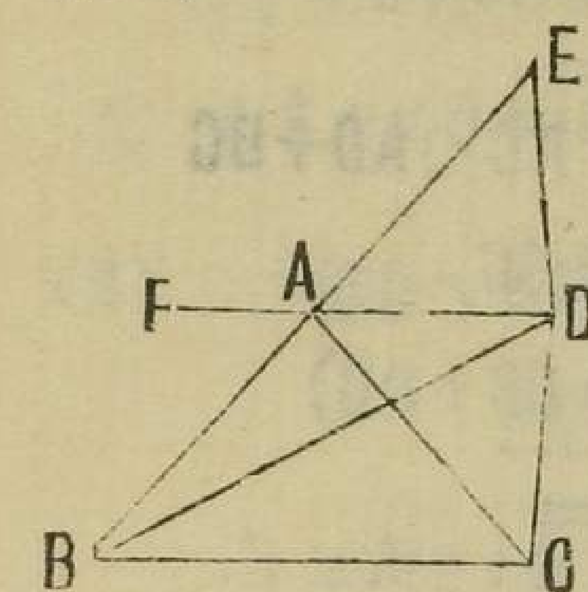
(42)



ABC ヲ二等邊三角形、
DBC ヲ同底同積ノ三角形、
$AE = AB$ トス

(証)

$\angle DAE = \angle FAB = \angle DAC \qquad AE = AC$

$\because \triangle DAE \cong \triangle DAC \quad \therefore DE = DC$

$\because BD + DE > BA + AE$

$\therefore BD + DC > BA + AC$

$\therefore BD + DC + CB > BA + AC + CB$

---

(43) (1) $\because AB + AC > BD + DC$

(証) $AB + BC > AD + DC$

$BC+AC > AD+BD$

$\therefore \ AB+AC+BC > AD+BD+DC$

(2) $\because \ AB < AD+BD \quad AC < AD+DC$

$BC < BD+DC$

$\therefore \ AB+AC+BC < 2(AD+BD+DC)$

(3) $\because \ AB+BE > AE \quad AC+CE > AE$

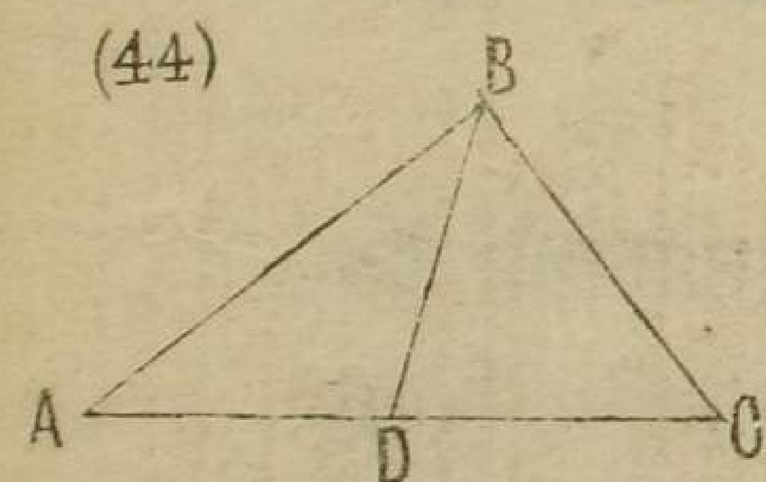
$\therefore \ AB+AC+BC > 2AE$

$AB+BC+AC > 2BF$

$BC+AC+AB > 2CG$

$\therefore \ AB+AC+BC > \frac{2}{3}(AE+BF+CG)$

(44)



ABCヲ三角形トス

(証)

(1) $\because \ BD=AD=DC$

$\therefore \ \angle A=\angle ABD$

$\angle C=\angle CBD$

$\therefore \ \angle A+\angle C=\angle ABC=$ r.∠.

(2) $\because \ BD > AD, DC \quad \therefore \ \angle A > \angle ABD$

$\angle C > \angle CBD$

$\therefore \ \angle A+\angle C > \angle ABC =$ 鋭角

(3) $\because \ BD < AD, DC \quad \therefore \ \angle A < \angle ABD$

$\angle C < \angle CBD$

$\therefore \ \angle A+\angle C < \angle ABC =$ 鈍角

(45) $\because \ \triangle ADB \cong \triangle ADE$

(証) $\therefore \ BD=DE$

(46) $\because \ BC \parallel EF, \ \therefore \ \angle BCE=\angle CEF=\angle ECF$

(証) $\therefore$ EF＝FC

$\because$ CD // FG, $\therefore$ ∠DCG＝∠CGF＝∠GCF

$\therefore$ FG＝FC＝EF

(47) $\because$ △ADG≌△BDC

(証) $\therefore$ ∠DAG＝∠DBC

同理ニテ ∠EAF＝∠BCE

$\therefore$ ∠DAG＋∠EAF＋∠BAC＝∠DBC＋∠BCE＋∠BAC＝2r.a.

故ニ FAG ハ一直線ナリ

(48) Dヲ貫キBCニ平行ニ引キタル線ヲABニEニACニFニ會セシム

(証) $\because$ BC // ED $\therefore$ ∠CBD＝∠BDE＝∠DBE

$\therefore$ BE＝ED

$\because$ ∠BAD＋∠ABD＝∠ADE＋∠BDE

$\therefore$ ∠BAD＝∠ADE $\therefore$ AE＝ED＝BE

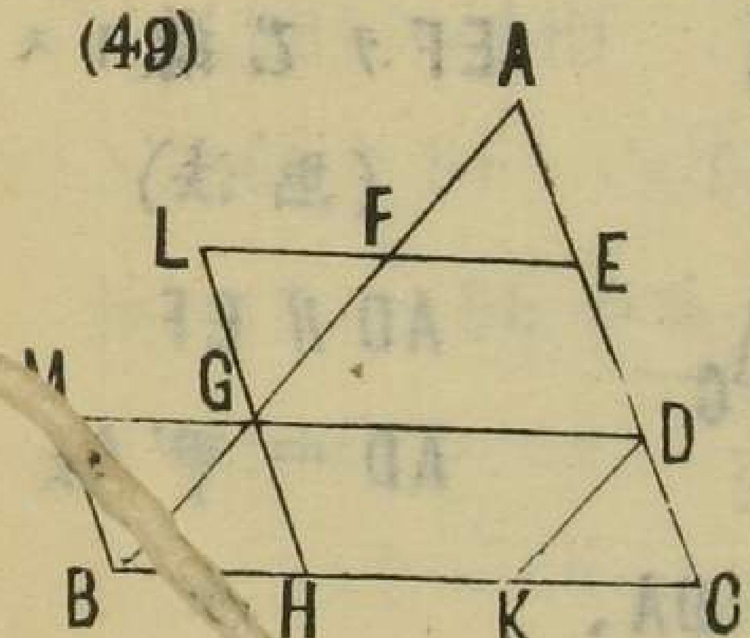
FEヲ延シBG // ACトシ以テAF＝FCナルヲ知ルベシ

ABCヲ三角形、

D, E, F, G, H, Kヲ等三分点、

GL // BM // ACトス

(49)



(証)

$\because$ ∠AFE＝∠GFL

∠FAE＝∠FGL

AF＝FG

$\therefore$ AE＝GL

$\because$ GL $\underline{\parallel}$ DE $\therefore$ LE // GD

$\because$ ∠GFL＝∠BGM　∠FGL＝∠GBM

$FG = GB \quad \therefore GL = BM = DE = DC$

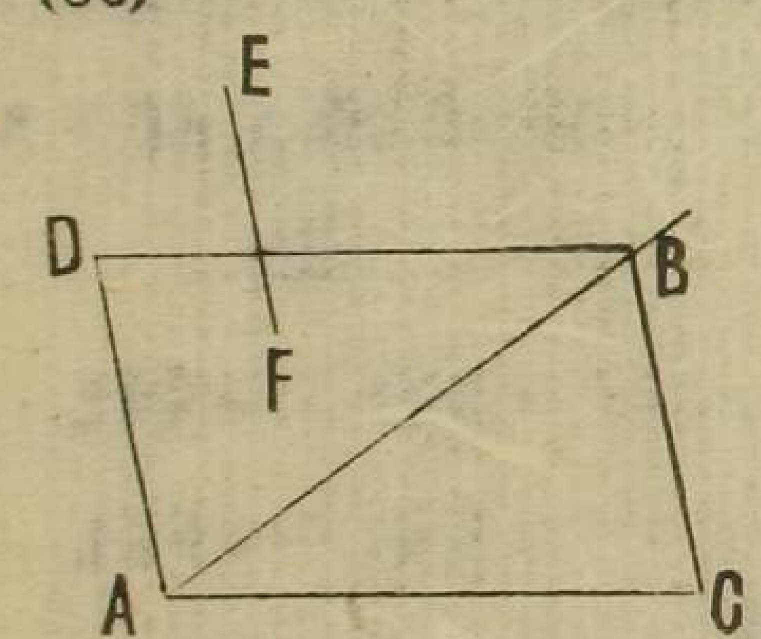
$\because BM \,\underline{\parallel}\, CD \quad \therefore MD \parallel BC \parallel LE$

故ニ$\triangle AFE$ト$\triangle ABC$ハ同形ナリ、

同理同法ヲ以テ$\triangle BGH$, $\triangle CDK$共ニ

$\triangle ABC$ト同形ナルヲ証スベシ

(50)



ABヲ甲

ACヲ乙

EFヲ乙線トス

(畫法)

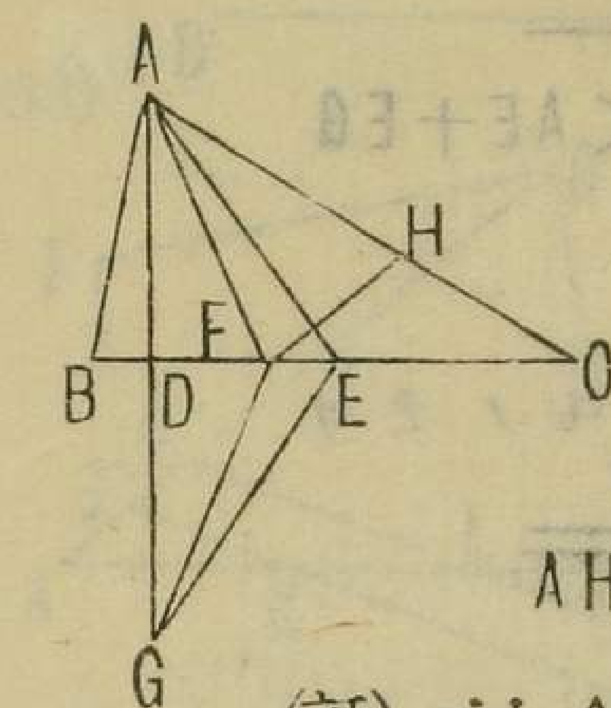
$AD \parallel EF$

AD＝甲線

$BD \parallel AC$、 $BC \parallel DA$、

BCハ求ムルモノナリ

(51)



ABCヲ三角形、

AFヲ∠Aノ折半線、

AEヲ底ノ折半線、

ADヲ底ノ垂線トス

(1) r.a.$>\angle B>\angle C$ノ片

$AH = AB \quad DG = AD$トス

(証) $\because \triangle ABF \cong \triangle AHF$

$\therefore \angle B = \angle AHF > \angle C \quad BF = FH$

$\because \angle CHF > \angle AHF \quad \therefore \angle CHF > \angle C$

$\therefore CF > HF \quad \therefore CF > FB < BE$

故ニFハBE中ニ在リ

$\because \angle BAF = \text{r.a.} - \frac{1}{2}(\angle B + \angle C)$

$\angle BAD = \text{r.a.} - \angle B$ 然レ片

$\because \angle B > \angle C, \quad 2\angle B > \angle B + \angle C$

$\angle B > \frac{1}{2}(\angle B + \angle C) \quad \therefore \angle BAF > \angle BAD$

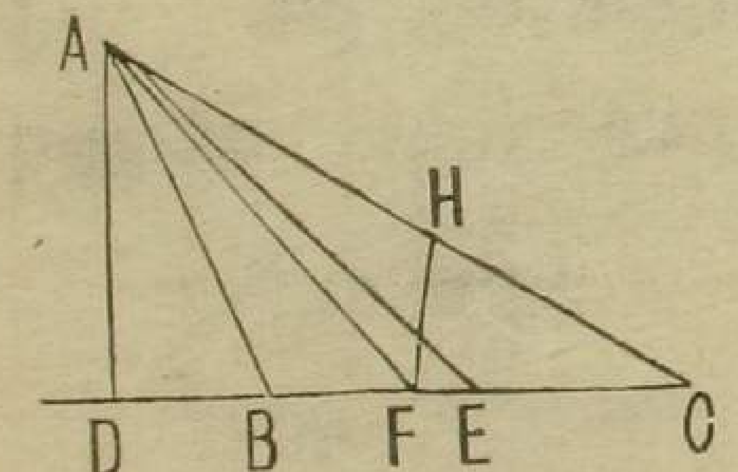
故ニFハDE中ニ在リ

∴ AG＜AF＋FG＜AE＋EG

∴ AD＜AF＜AE

故ニAF亦中間ノモノナリ

(2)



∠B＞r.a.ノ時

AH＝ABトス

(証)

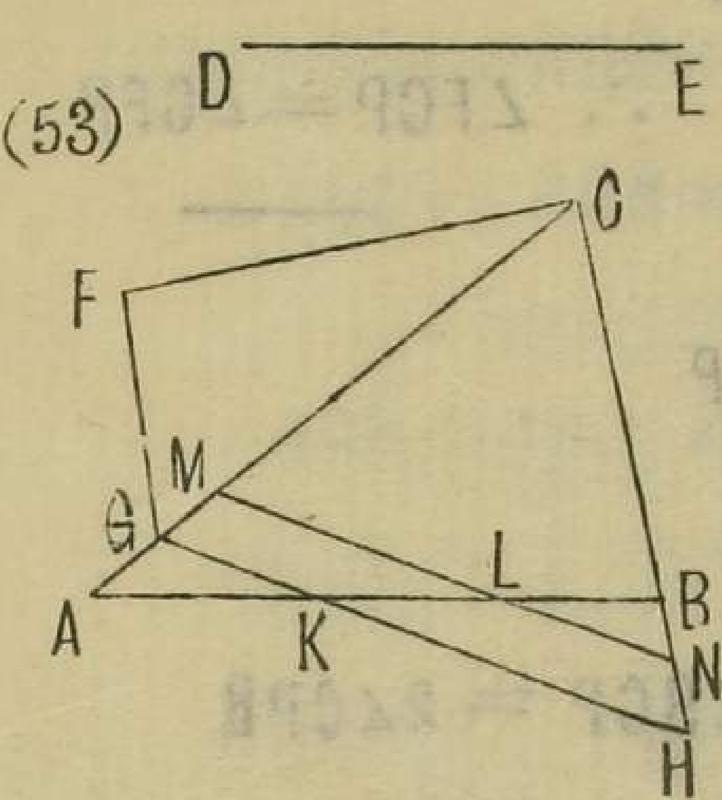
∵ △ABF≌△AHF

∴ ∠ABD＝∠CHF

∵ ∠ABD＞∠C　∴ ∠CHF＞∠C

(52) (画法) 項角ノ拆半線ガ底ニ會スルノ点ハ即求ムル所ノモノナリ

(53)



ABCヲ三角形、DEヲ固有線トス

(1) (画法)

CF⊥CB

CF＝DE

FG // CB

CH＝CG

Kハ求ムル点ナリ

∠C鈍角ナルモ亦同法ナリ

(2) CM＝CN＝DE

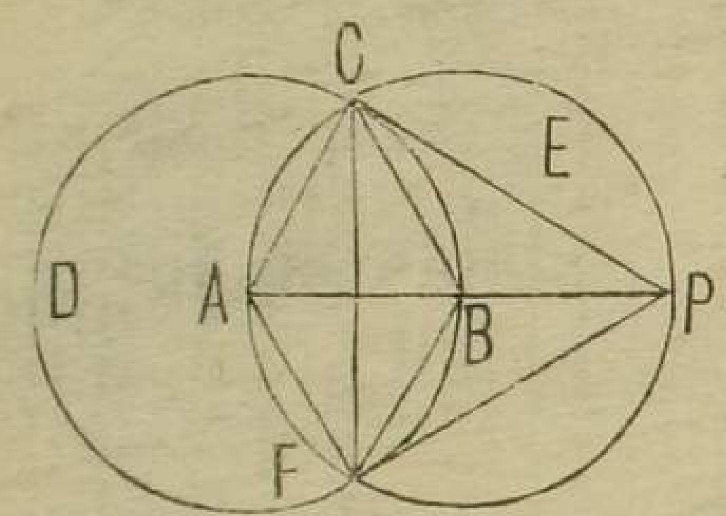
[illegible]ハ求ムル点ナリ

ABヲ前知線トス

(54) (証)

$\because\ \triangle CBP \cong \triangle FBP,\quad \therefore\ CP = FP$

$\therefore\ \angle FCP = \angle CFP$



$\because\ \angle ABC = \angle CPB + \angle BCP = 2\angle CPB$

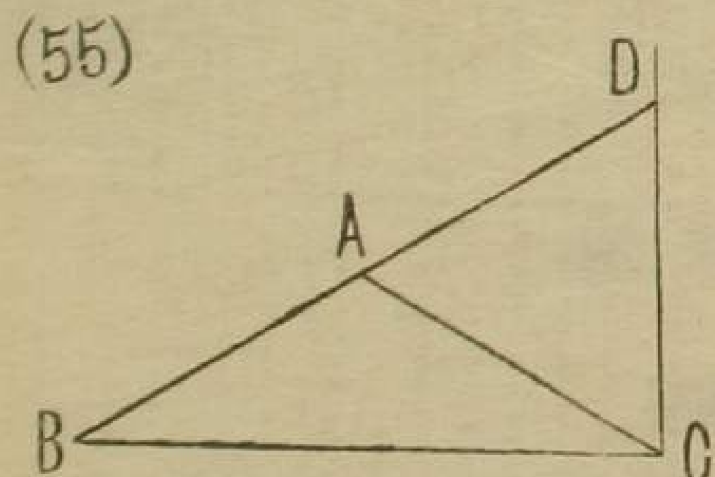
$\therefore\ \angle CPB = \frac{1}{2}\angle ABC = \frac{1}{3}$ r.a.

同理ニテ $\angle FPB = \frac{1}{3}$ r.a. $\therefore \angle CPF = \frac{2}{3}$ r.a.

$\therefore\ \angle PCF = \angle PFC = \frac{2}{3}$ r.a.

故ニ △CPF ハ等角ナレハ等邊ナリ

(55)



ABC ヲ二等邊三角形 $CD \perp BC$ トス

(証)

$\because\ \angle ABC = \angle ACB = \frac{1}{4}\angle BAC = \frac{1}{6}\, 2$r.a.

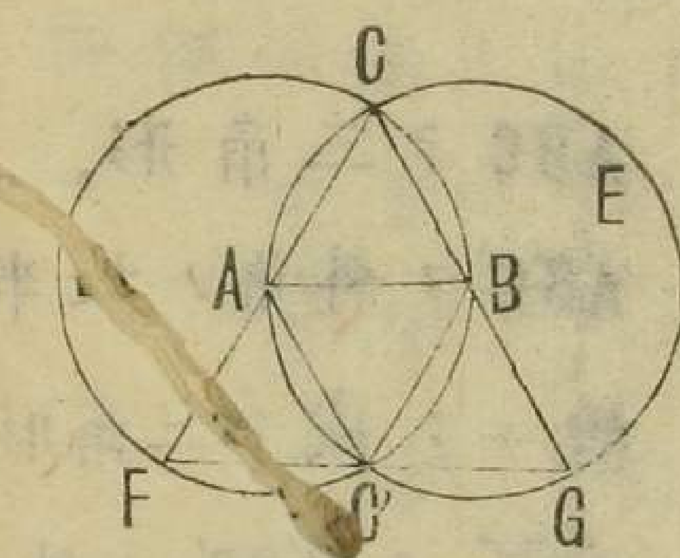
$\therefore\ \angle ABC + \angle ACB = \frac{2}{3}$ r.a. $= \angle CAD$

$\because\ \angle ACD =$ r.a. $- \angle ACB = \frac{2}{3}$ r.a.

$\therefore\ \angle ADC = \frac{2}{3}$ r.a. 故ニ △ACD ハ等邊也

(56) $\because\ \angle CAB = \angle BAC' = \frac{2}{3}$ r.a.

(証) $\therefore\ \angle FAC' = \frac{2}{3}$ r.a.



$\because\ AF = AC'$

$\therefore\ \angle AFC' = \angle AC'F = \frac{2}{3}$ r.a.

同理ニテ

$\angle BC'G = \frac{2}{3}$ r.a.

又 $\angle AC'B = \frac{2}{3}$ r.a.

$\therefore\ \angle AC'F + \angle BC'G + \angle AC'B = 2\,\mathrm{r.a.}$

故ニ FC'G ハ一直線ナリ又△C'FG ハ等角ナレバ等邊ナリ

---

(57) $\because\ 2\angle CBD = 2\,\mathrm{r.a.} - \angle ABC$

(証) $2\angle BCD = 2\,\mathrm{r.a.} - \angle ACB$

$\therefore\ \angle CBD + \angle BCD = 2\,\mathrm{r.a.} - \frac{1}{2}(\angle ABC + \angle ACB)$

$\therefore\ \angle D = \frac{1}{2}(\angle ABC + \angle ACB)$　$\frac{1}{2}A$ ヲ加フレバ $\angle D + \frac{1}{2}\angle A = \mathrm{r.a.}$

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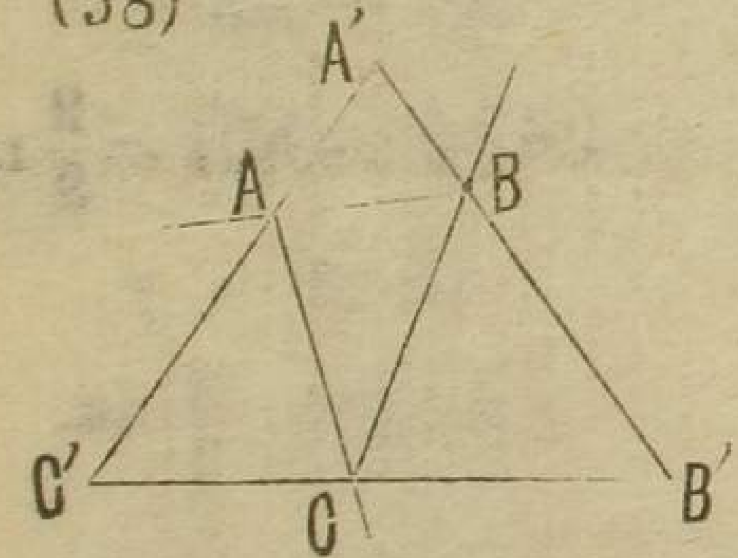
(58)



ABC ヲ三角形、A'B'C' ヲ外角ノ折半線ニテ成ス三角形、A''B''C'' ヲ A'B'C' ノ外角ノ折半線ニテ成ス三角形 餘之レニ倣ヘ、此証中角符∠ヲ省ク

(証)

$\therefore\ A' = \frac{1}{2}(A+B)$　　$B' = \frac{1}{2}(B+C)$

$C' = \frac{1}{2}(A+C)$ ---------(57)

$\therefore\ A' - B' = \frac{1}{2}(A-C)$ --------(1)

$A' - C' = \frac{1}{2}(B-C)$ --------(2)

$C' - B' = \frac{1}{2}(A-B)$ --------(3)

$\because\ A'' = \frac{1}{2}(A'+B') = \frac{1}{4}(A+2B+C)$

同理同法ヲ以テスレバ

$\therefore\ C'' - B'' = \frac{1}{4}(A-C)$ ------(4)

$A'' - B'' = \frac{1}{4}(B-C)$ -------(5)

$C'' - A'' = \frac{1}{4}(A-B)$ -------(6)

---

(1) (2) 等ヲ比較シ考フルニ第二三角

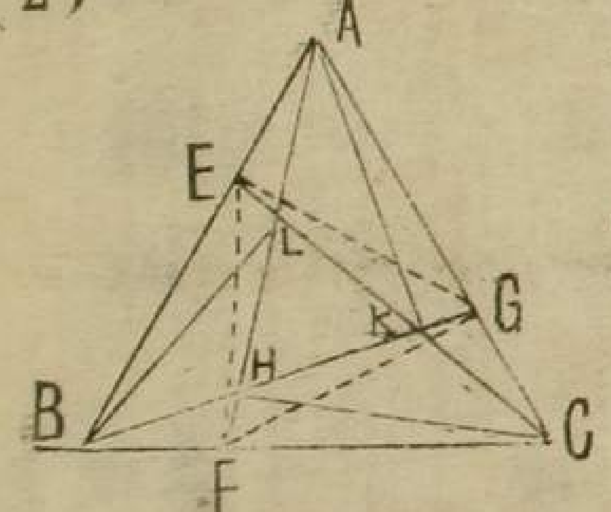
形ノ各兩角ノ差ハ第一三角形ノ各兩角ノ差ノ半、第三ノ各兩角ノ差ハ第一ノ各兩角ノ差ノ四分一、次ハ八分一 十六分一等ニテ三角形ノ數増加スルニ從フテ其角ノ差彌々減少シ漸次零ニ近ヅキテ即等辺三角形ヲ成サントス

(59) (1) ∵ △BEF≌△AGE

(証) ∴ EF＝EG　同理ニテ

EF＝FG　∴ EF＝EG＝FG

(2)



∵△ABF≌△BCG

∴∠BAF＝∠CBG

∠AFB＝∠CGB

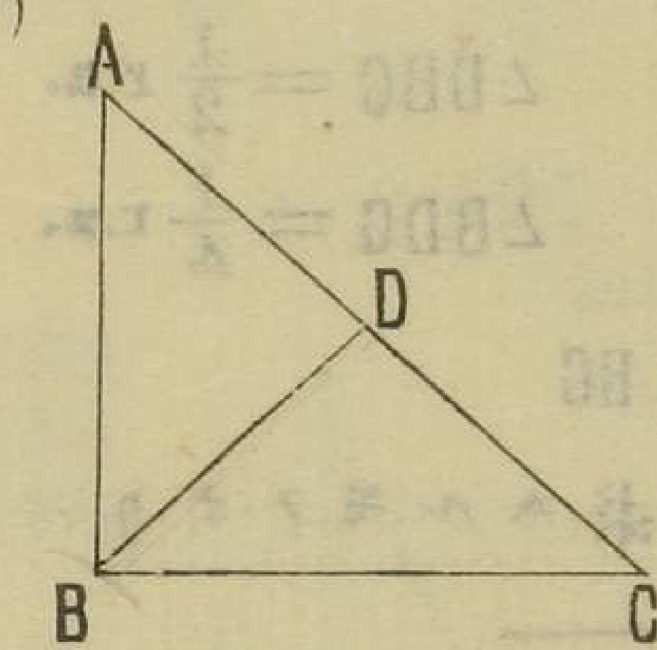
同理ニテ

∠BAF＝∠ACE　∠AFB＝∠CEA

∴ ∠LHK＝∠KLH＝∠HKL

∴ LHK ハ等邊三角形

(60)



(畫法)

BD＝弦ノ垂線、

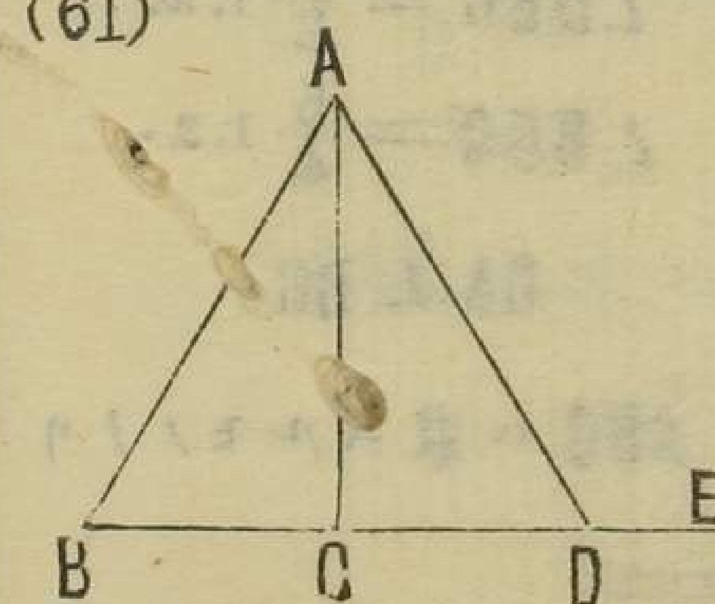
ADC⊥BD、

BC＝底

BA⊥BC

ABC ハ求ムルモノナリ

(61)



(畫法)

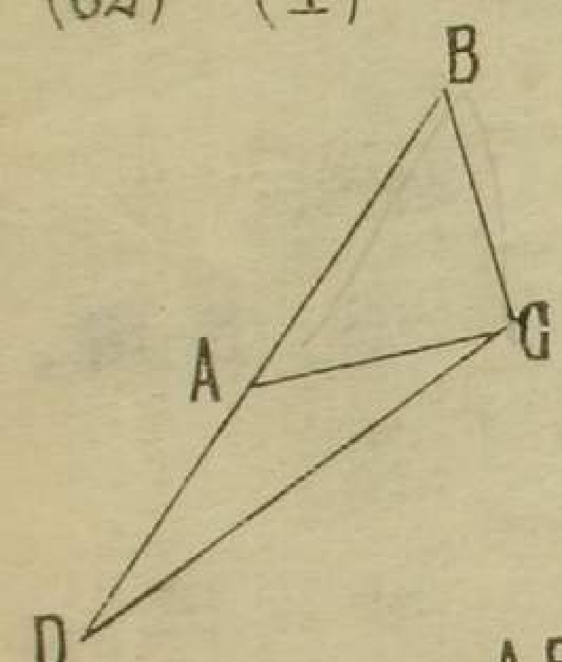
ED ヲ二邊ノ和ト弦ノ差、EC ヲ一邊、

CA⊥CE, CA＝CE,

∠DAB＝∠ADC

ABCハ求ムルモノナリ

(62) (1)



(画法)

BD＝弦ト一邉ノ和

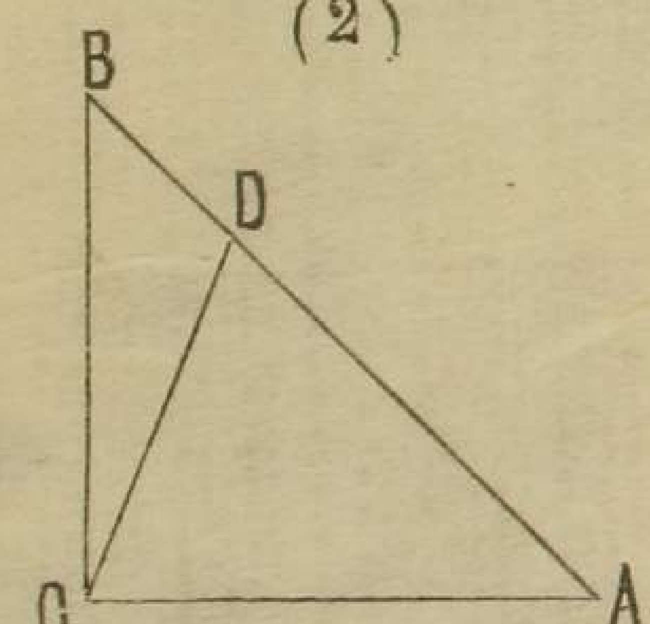
$\angle DBC = \frac{1}{2}$ r.a.

$\angle BDC = \frac{1}{4}$ r.a.

CA ⊥ BC

ABCハ求ムルモノナリ

(2)



BD＝弦ト一邉ノ差

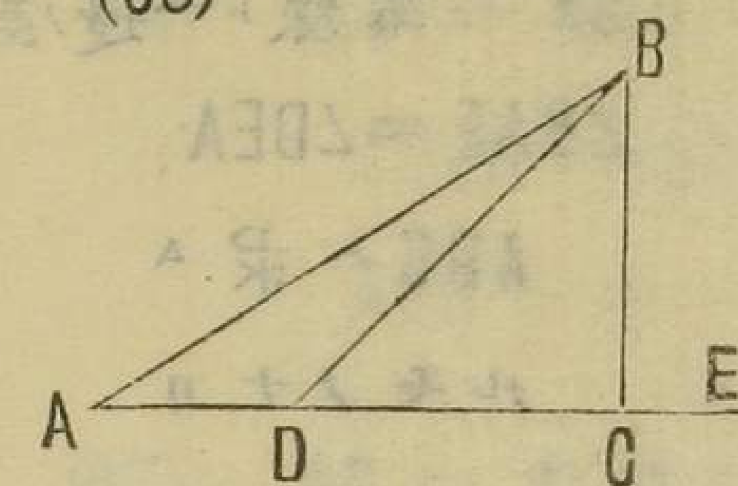
$\angle DBC = \frac{1}{2}$ r.a.

$\angle BDC = \frac{5}{4}$ r.a.

CA ⊥ BC

ABCハ求ムルモノナリ

(63)



(画法)

AD＝二邉ノ差、

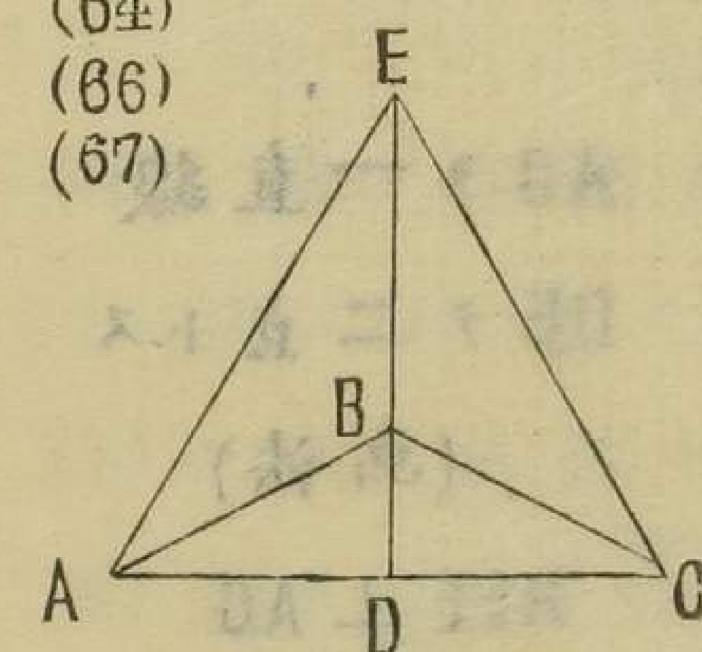
AE＝弦

$\angle BDC = \frac{1}{2}$ r a

AB＝AE　BC ⊥ AE

ABCハ求ムルモノナリ

(64)
(66)
(67)



(画法)

(1)　AC＝底

AD＝DC

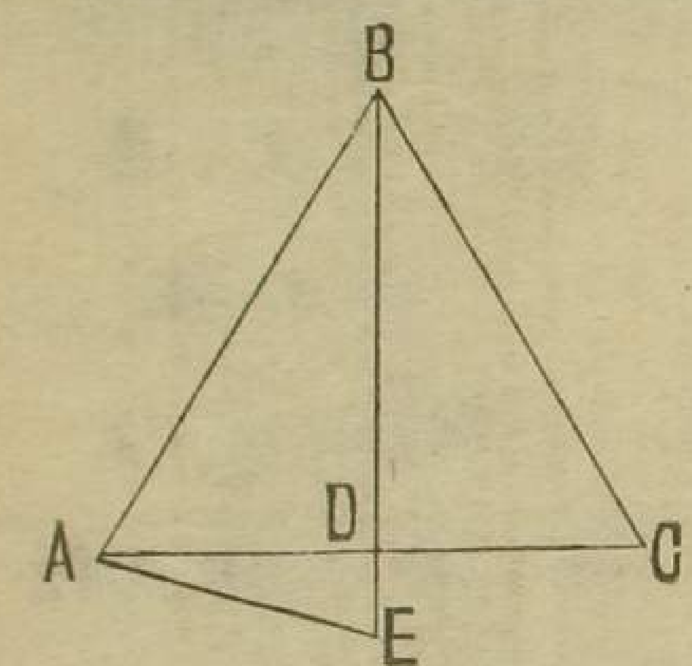
DE ⊥ AC

DE＝垂線ト一辺ノ和

∠BAE＝∠AEB

ABCハ求ムルモノナリ

(2)



DE＝垂線ト一邊ノ差

∠BAE＝∠DEA

ABCハ求ムルモノナリ

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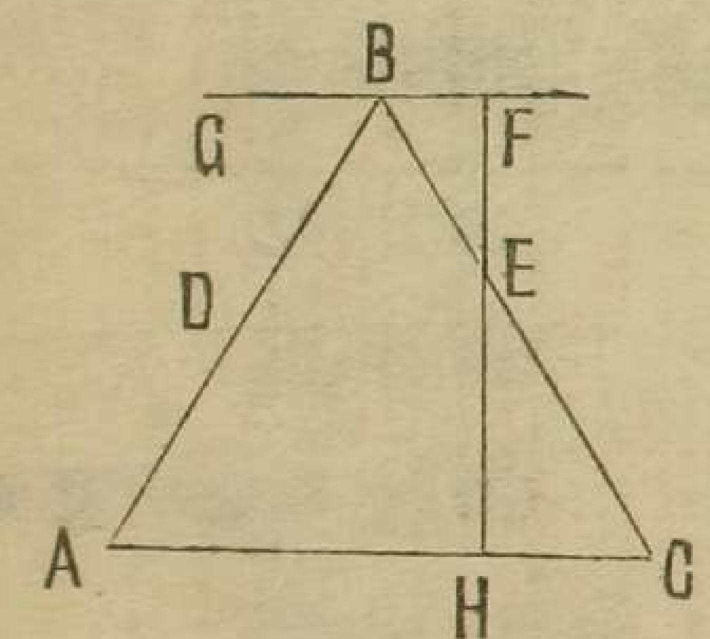
(65)



ACヲ一直線

D,Eヲ二点トス

(画法)

HEE ⊥ AC

HF＝高

FBG // CA　∠EBF＝∠DBG ……(4)

ABCハ求ムルモノナリ

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(66) ED＝角点ヨリノ垂線、

$\angle DEA = \angle DEC = \frac{1}{3}$ r.a.　ADC ⊥ DE

AECハ求ムルモノナリ

---

(67) AB＝底角ノ折半線、

$ABC = \frac{4}{3}$ r.a.　BC＝BA

ACハ求ムルモノナリ

---

(68) 法ニ二種アリ (1)ハ二邊挟角 (2)ハ二邊一對角ナリ

(1)ハ別ニ画法ヲ説ク迄モナシ

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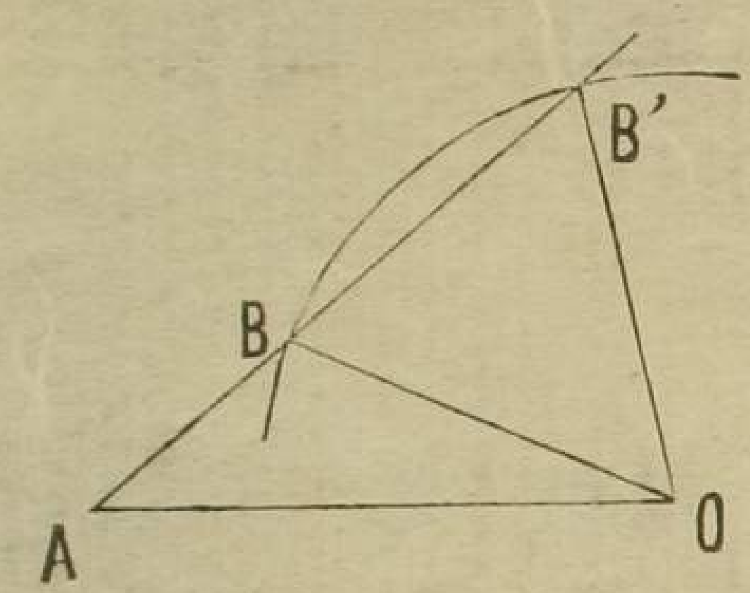
(2) 角小邊ニ對シテ鋭角ナルトキハ不定ニシテ問ニ應スル三角形二個ヲ得ベシ

(証)

AC＝大邊　B'AC＝小邊ニ對スル鋭角トス



今Cヲ中心トシ小邊ノ長サヲ半徑トシテ圈ヲ画ケバB及B′ニ於テAB′ヲ貫クベケレバABC、AB′Cノ両三角形ハ二邊一對角ヲ等フス故ニ共ニ問ニ應スルモノナリ

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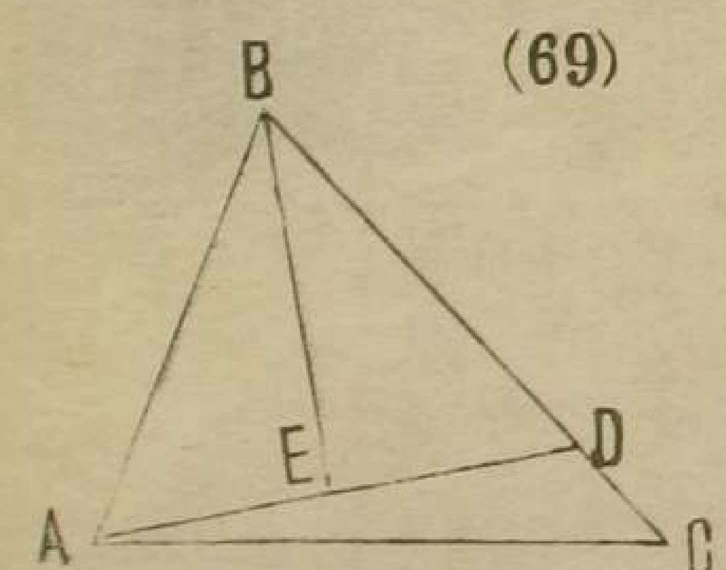
(69)



(画法)

AC＝底

∠CAD＝底角ノ差半、

DC＝二邊ノ差、

AE＝ED, EB⊥AD,

ABCハ求ムルモノナリ

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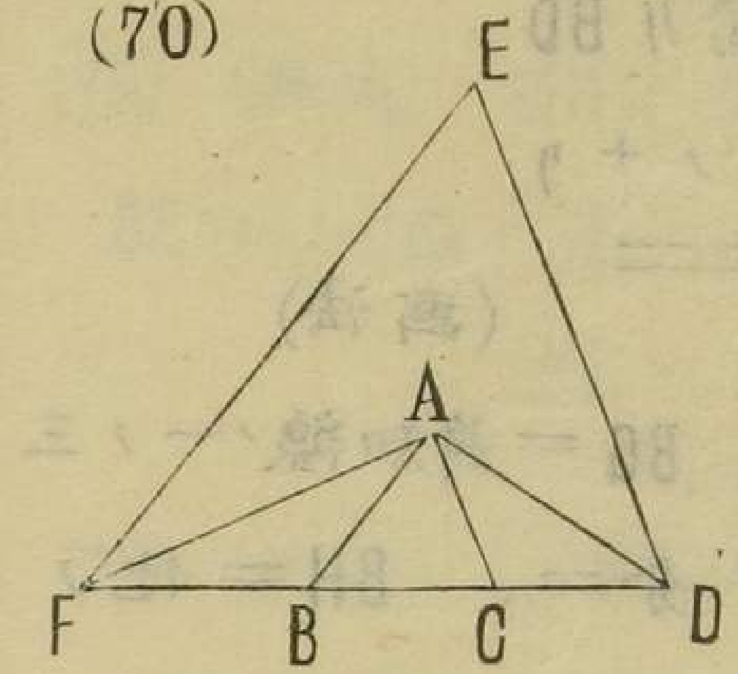
(70)



(画法)

FD＝周圍、

∠DFE＝一角、

∠EDF＝他ノ角

∠DFA＝∠EFA

∠FDA＝∠EDA

AB∥EF、AC∥ED、

ABCハ求ムルモノナリ

---

(71) (画法) BC＝底 ∠BCA＝底角ノ和半＋其差半、∠CBA＝底角ノ和半－其差半、ABCハ求ムルモノナリ

---

(72) BD, CDヲ二線、Aヲ其間ニ在ル一

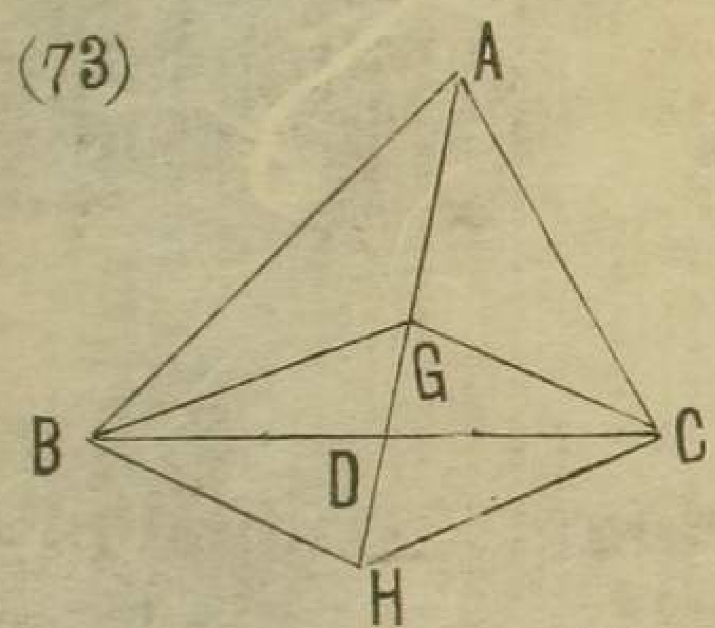
点トス

(画法)　AB∥CD、AC∥BD

ABCハ求ムルモノナリ

(画法)

(73)



BG＝前知線ノ一ノ三分二、BH＝他ノ一ノ三分二、GH＝又他ノ三分二

GD＝DH、DC＝DB、GA＝GH、ABCハ求ムルモノナリ

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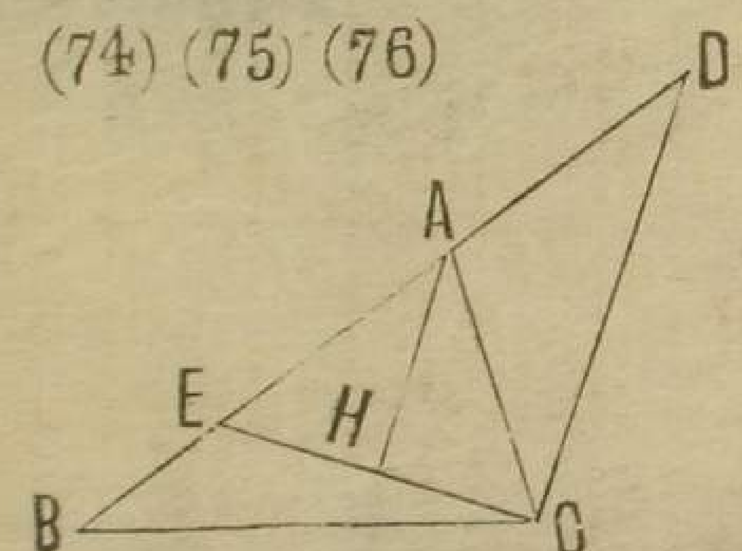
(74) (75) (76)



(画法)

BC＝底

∠CBD＝底角

BD＝二邊ノ和

∠DCA＝∠BDC、ABCハ求ムル者也

---

(75) (画法) BC＝底、∠CBA＝底角、BE＝二邊ノ差、EH＝HC、HA⊥EC、

ABCハ求ムルモノナリ

---

(76) (画法) BE＝二邊ノ差

∠BEC＝r.a.＋對角ノ半、BC＝一邊、EH＝HC、AH⊥EC

ABCハ求ムルモノナリ

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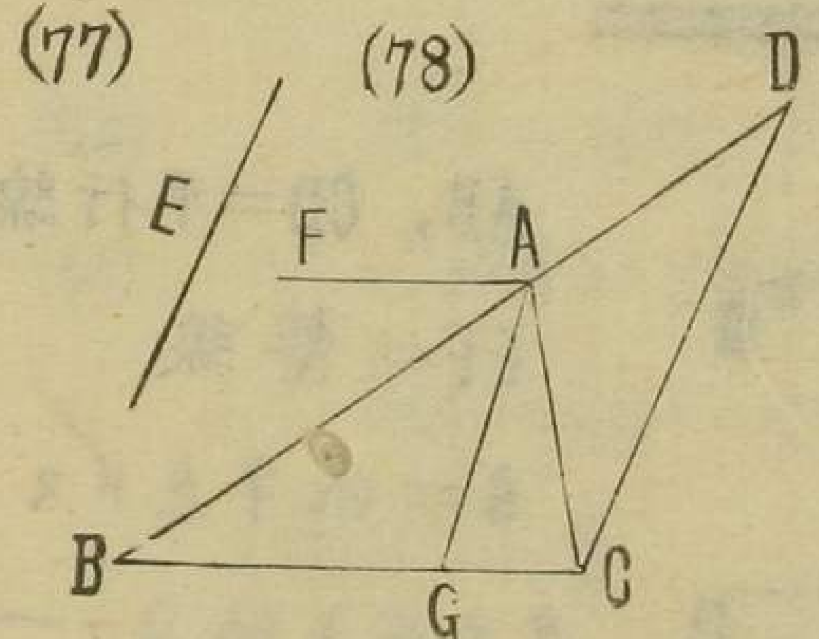
(77) (78)



Eヲ預設ノ一直線トス

(画法)

BC＝底, CD∥E

BD＝二邊ノ和

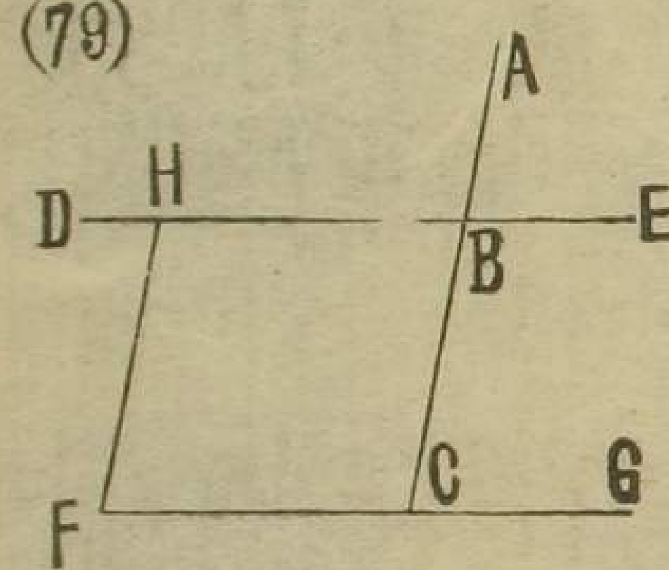
$\angle DCA=\angle BDC$, ABCハ求ムル者也

(78) BC＝一直線, A＝設点

(画法) $AF \parallel BC$, $\angle FAG$＝前知角、

AGハ求ムルモノナリ

(79)



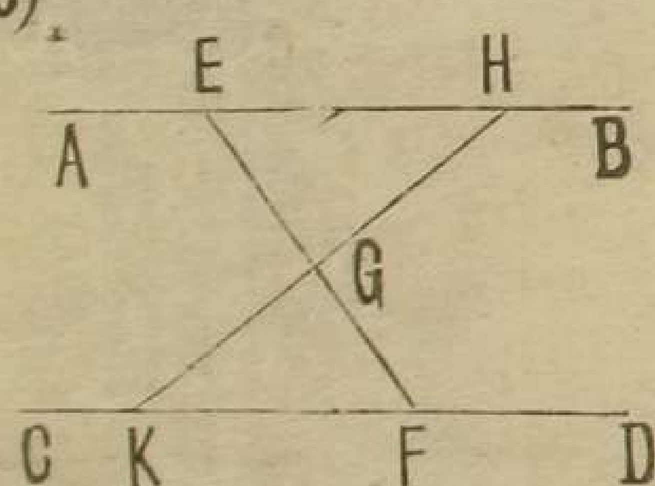
DE, FG＝平行線

(画法)

FH＝前知線

$ABC \parallel HF$

(80)



AB, CD＝平行線

EF＝繋線

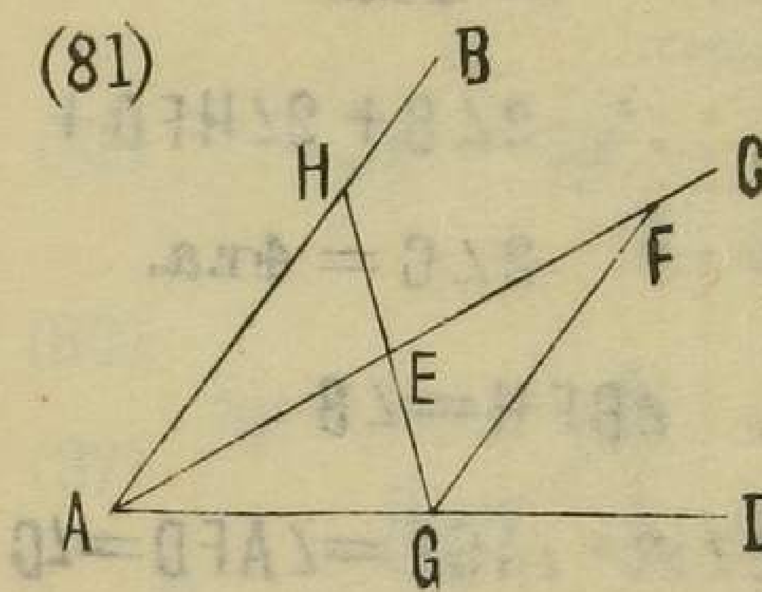
G＝折半点トス

Gヲ串キ随意ノ一線ヲ引キHGKト命ス

(証) $\because \angle GEH=\angle GFK$　$\angle EGH=\angle FGK$

$EG=GF$, $\therefore GH=GK$

(81)



AB, AC, ADヲ三線トス

(画法)

AC中ニ随意ニE点ヲ設ケ、$AE=EF$, $FG \parallel AB$

GEHハ求ムルモノナリ

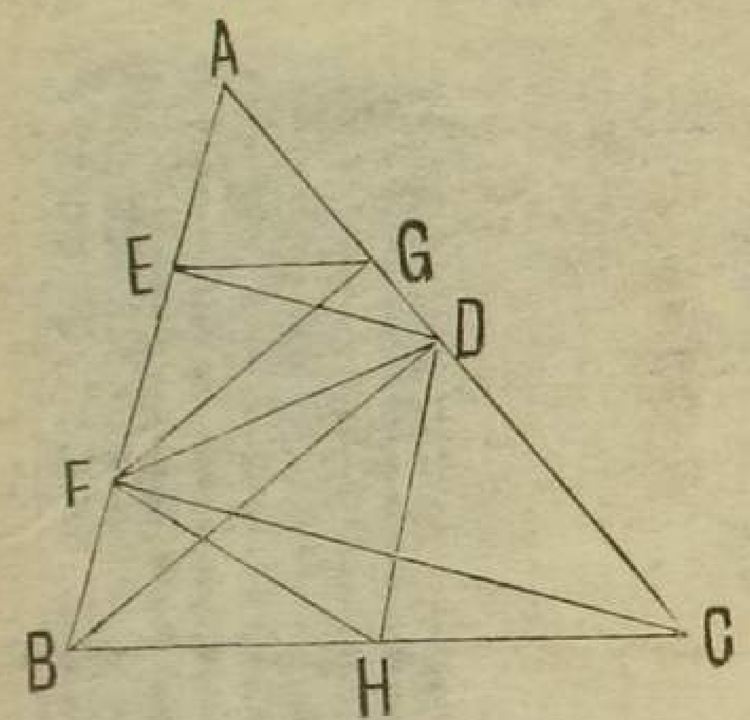
(82) $BH=HC$トス

(証) $\because \angle BFC=\angle BDC=$ r.a.

$\therefore BH=FH=HD=HC$ ····· (32)

$\therefore \angle BFH=\angle B$　$\angle CDH=\angle C$

$\angle HFD=\angle HDF$



$\because \angle B+\angle BFH+\angle HFD+\angle HDF+\angle HDC+\angle C = 4r.a.$

$\therefore 2\angle B+2\angle HFD+2\angle C = 4r.a.$

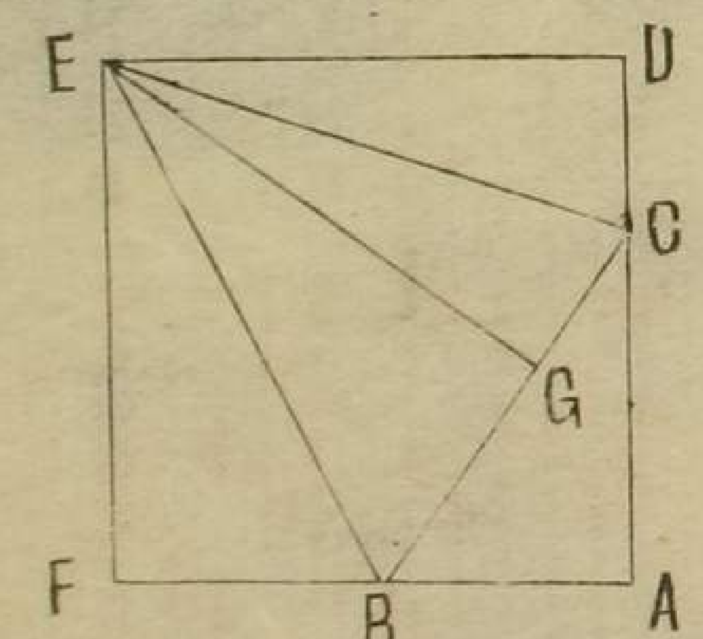
$\therefore \angle HFD=\angle A$ 又 $\angle BFH=\angle B$

$\therefore \angle AFD=\angle C$ 同理ヲ以テ$\angle AGE=\angle AFD=\angle C$

$\therefore EG \parallel BC$

---

(83)



$EG \perp BC$ トス

(証)

$\because \triangle EBF \cong \triangle EBG$

$\therefore EF = EG$

同理ヲ以テ

$ED = EG = EF$

$\because \angle A = \angle D = \angle F = r.a. \therefore \angle DEF = r.a.$

$\therefore$ ADEF ハ正方形トク

(84) (画法) 此二對ヲ以テ矩形ヲ画クベシ

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(85) 此題ノ解ハ巻末ニ掲グ

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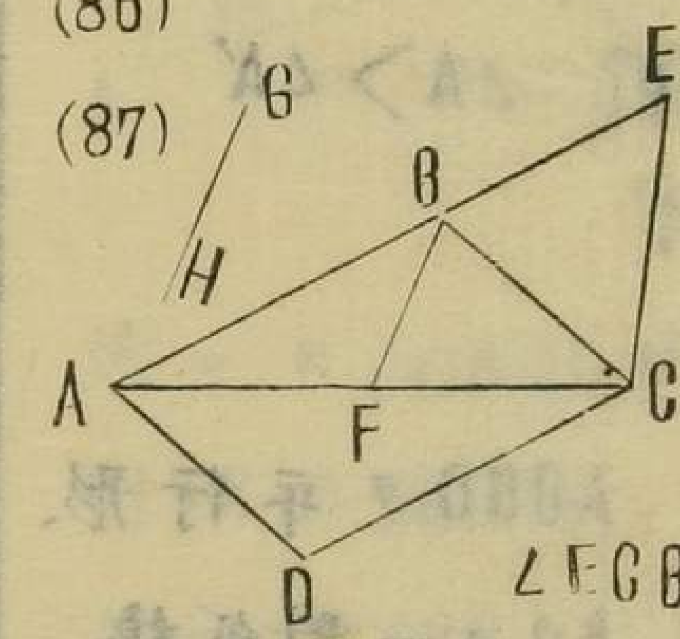
(86)

(87)



(画法)

AE = 相隣邊ノ和、

$\angle AEC$ = 挾角ノ半

AC = 對角線、

$\angle ECB=\angle AEC$, AD = BC,

CD = AB, ABCD ハ求ムルモノナリ

---

(87) (画法) AC = 對角線

$\angle CAB$ = 前知角 AF = FC、

$FB \parallel$ 預定ノ傍線 GH、 AD = BC、

$CD=AB$、 ABCD ハ求ムルモノナリ

(88) (証) $\because \angle A+\angle B+\angle C+\angle D=\angle A'+\angle B'+\angle C'+\angle D'$

然レ𪜈 $\angle A=\angle C>\angle A'=\angle C'$

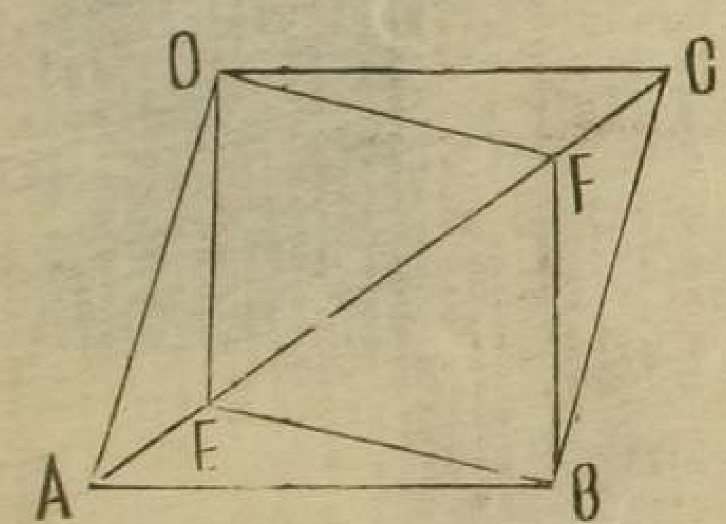
$\therefore \angle B=\angle D<\angle B'=\angle D'$ 又 $\because AD=A'D'$、

$DC=D'C'$、 $\angle D<\angle D'$、 $\therefore AC<A'C'$

$\because AD=A'D' \quad AB=A'B' \quad \angle A>\angle A'$

$\therefore BD>B'D'$

(89)



ABCD ヲ平行形、

AC ヲ一對角線、

$AE=FC$ トス

(証)

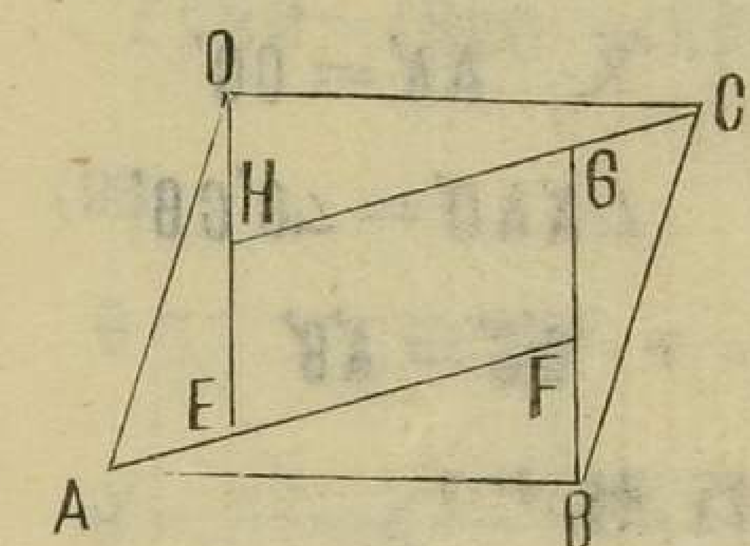
$\because AD=BC$

$AE=CF \quad \angle DAE=\angle BCF \quad \therefore DE=BF$

同理ニテ $DF=BE$ 對邊相同ケレバ

BEDF ハ平行形ナリ

(90)



ABCD ヲ平行形

$\angle BAF=\angle CBG=$

$\angle DCH=\angle ADE$ トス

(証)

$\because \angle DEF=\angle DAE+\angle ADE$

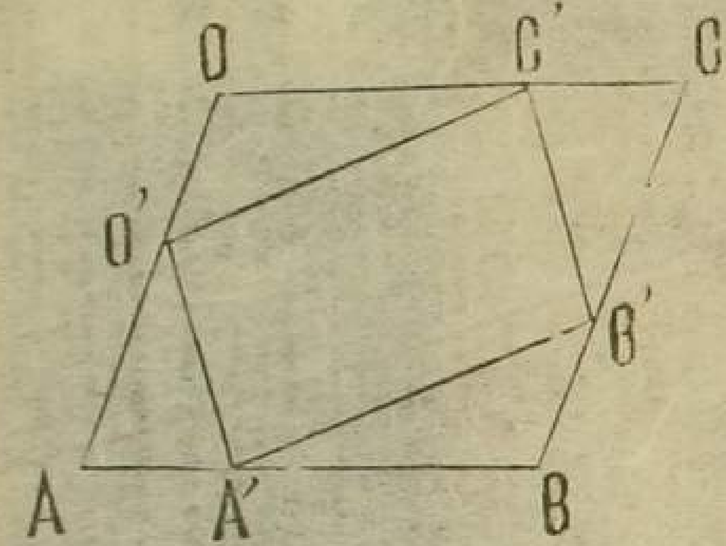
然レ𪜈 $\angle ADE=\angle BAF$, $\therefore \angle DEF=\angle BAD$、

同理同法ニテ $\angle CHE=\angle ADC$

$\angle AFG=\angle ABC \quad \angle BGH=\angle BCD$ ヲ知ル

$\therefore$ EFGH ハ ABCD ト等形ニシテ平行形ナリ

(91) ABCD ヲ平行形トス



(証)

$\because AO = CB$

$OO' = BB'$, $\therefore AO' = CB'$

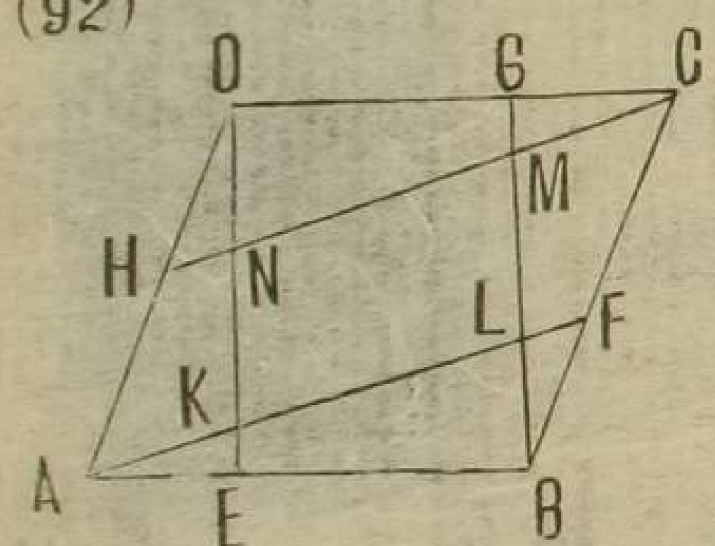
又 $AA' = CC'$

$\angle A'AO' = \angle C'CB'$

$\therefore A'O' = C'B'$ 同法ニテ $O'C' = A'B'$

$\therefore A'B'C'O'$ ハ平行形ナリ

(92)



(証)

$\because AH \,\underline{\parallel}\, GF$

$\therefore AF \,\underline{\parallel}\, GH$

同理ニテ $OE \,\underline{\parallel}\, GB$

$\therefore KLMN$ ハ平行形ナリ

$\because \angle ABC = \angle ABG + \angle GBC = \angle BGC + \angle GBC$

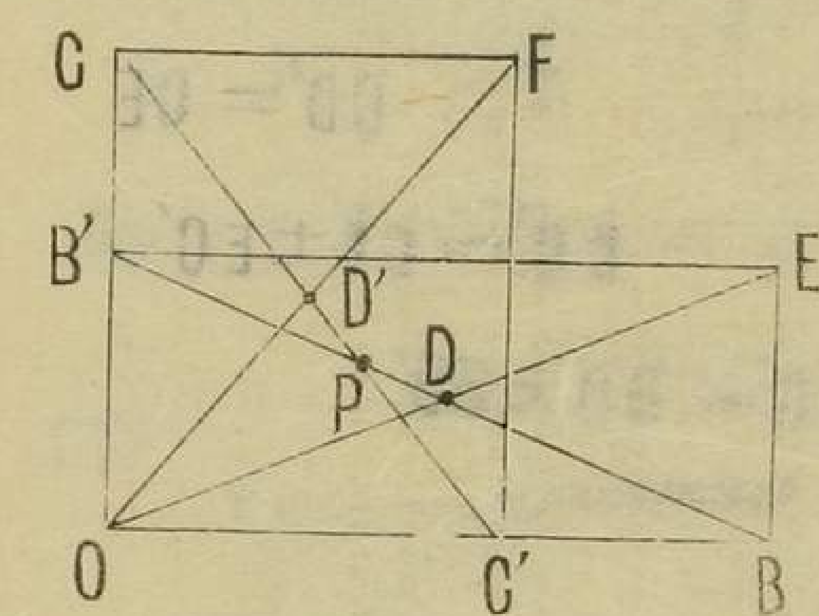
$\angle LMN = \angle ALB = \angle AFB + \angle GBC$

$\therefore \angle ABC - \angle LMN = \angle BGC - \angle AFB$

$\because \angle KNM + \angle LMN = \angle ABC + \angle BCD$

$\therefore \angle KNM - \angle BCD = \angle ABC - \angle LMN = \angle BGC - \angle AFB$

(93)



$BE \perp OB$

$C'F \perp OB$ トイレバ

$OBEB'$、$OC'FC$ ハ

矩形ナリ

(証)

$\because \angle FOC' = \angle CC'O$, $\angle EOB = \angle B'BO$

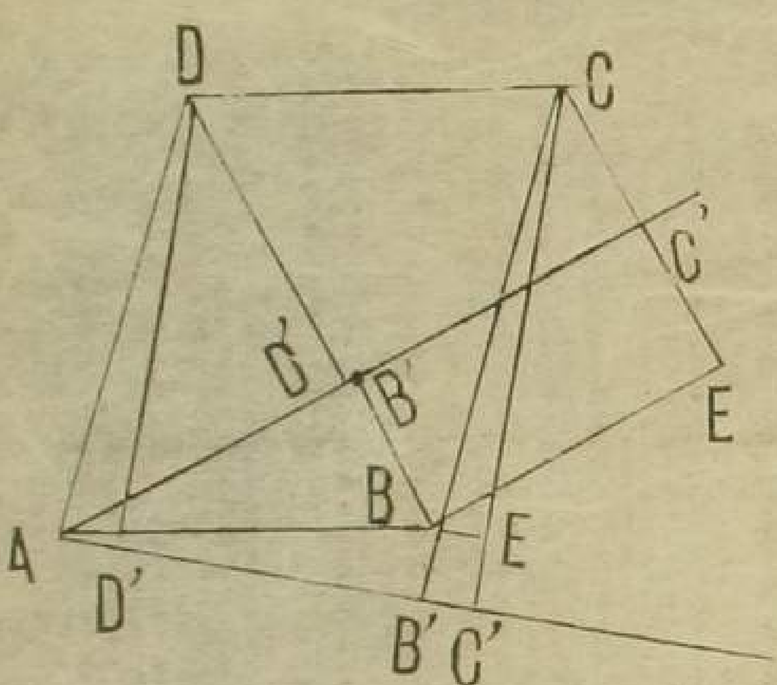
$\therefore \angle FOC' - \angle EOB = \angle CC'O - \angle B'BO$

$\therefore \angle DOD' = \angle BPC' = \angle B'PD'$

(94) AC' ヲ随意線

CC', BB', $DD' \perp AC'$、$BE \parallel AC'$ トス

(証)



$\because$ AD ∥ BC

BE ∥ AC′

$\therefore \angle DAD' = \angle CBE$

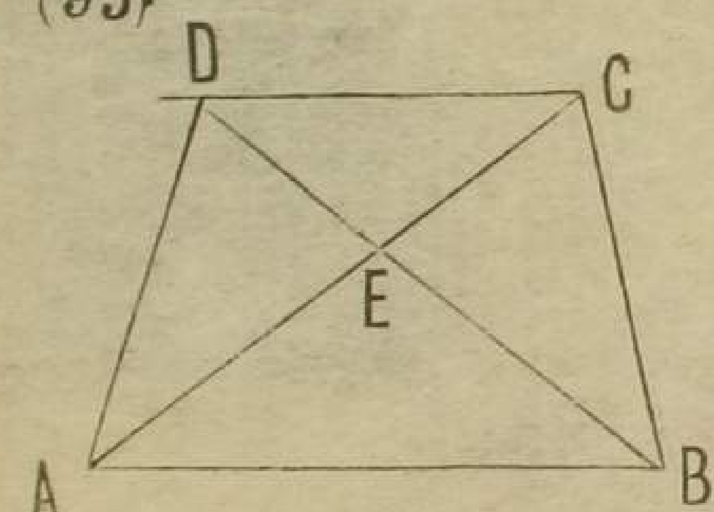
$\because \triangle ADD' \cong \triangle BCE$

$\therefore DD' = CE$

又 $BB' = C'E$　　$CC' = CE \pm EC'$

$\therefore CC' = DD' \pm BB'$

(95)



(証)

$\because \angle ACD = \angle BDC$

$\angle DAC = \angle CBD$

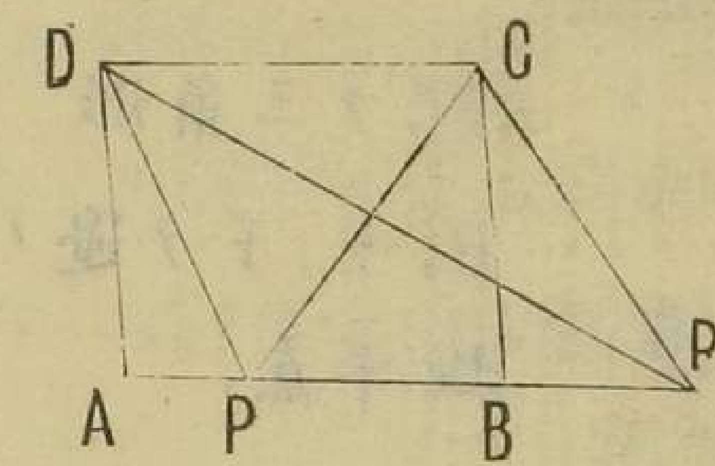
$DC = DC, \therefore AC = BD$

$\because DE = EC$　$\therefore BE = EA$

$\therefore \angle EAB = \angle EBA$　又 $\angle ECD = \angle EDC$

$\therefore \angle EBA = \angle EDC$　$\therefore$ AB ∥ DC

(96)



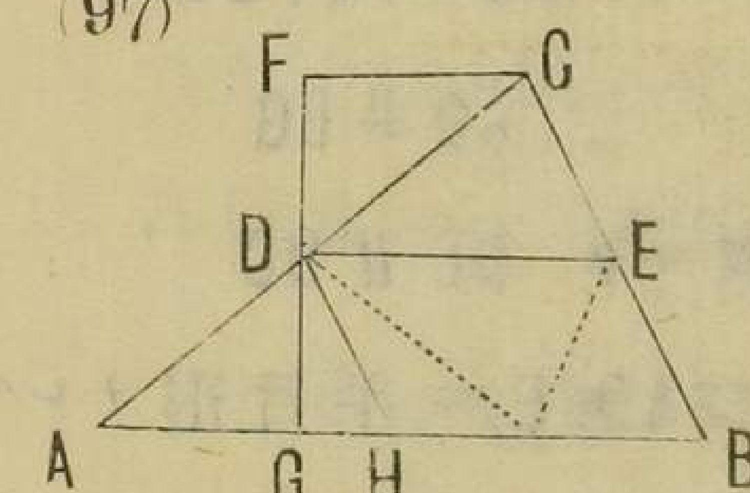
ABCDヲ平行形トス

(畫法)

$CP = CD$

DPハ∠APCノ折半線ニテPハ求ムル點ナリ

(97)



邊ヲ延シ反セバ三角形ABCトナリ、CDEハ第一變、DABEハ第二變トナリ、

FDG ⊥ AB，　DH ∥ BE，　CF ∥ AB トス

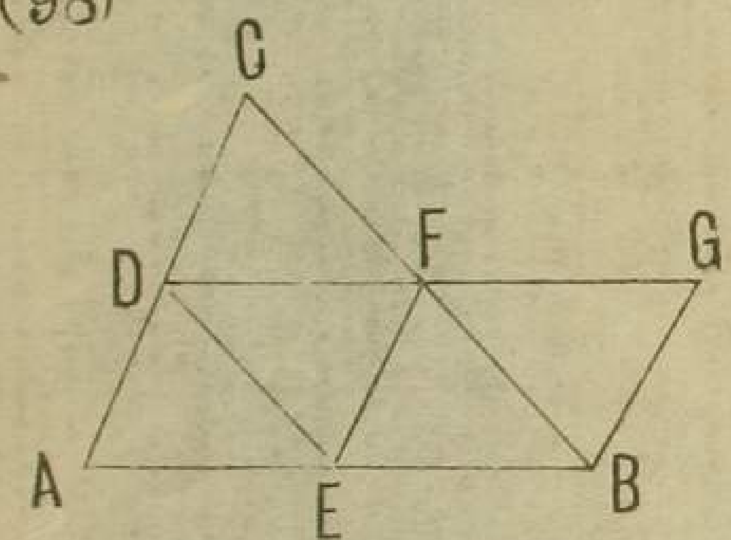
(証)　$\because DF = DG$　　$\angle CDF = \angle ADG$

$\angle DFC = \angle DGA$　　$\therefore CD = DA$

∵ △ADH＝△CDE 又 ▱DHBE＝2△CDE

∴ ⏢DABE＝3△CDE

(98)



ABCヲ三角形

D, E, Fヲ邊ノ

折半点、

FG＝FDトス

(証)

∵ △CFD≌△BFG　∴ ∠C＝∠FBG

∴ CD $\stackrel{\parallel}{=}$ BG＝AD　∵ AD $\stackrel{\parallel}{=}$ BG

∴ DG $\stackrel{\parallel}{=}$ AB　同理ニテ DE // BC

EF // AC, ∵ ▱ADFE＝平行形ナレバ

∴ △DEF＝△ADE

同理ニテ △DEF＝△CDF＝△BEF

∴ △DEF＝$\frac{1}{4}$△ABC

(99) (証) ∵ AE＝ED ∴ △AEC＝△CED

△AEB＝△BED ∴ △AEC＋△AEB＝△CED＋

△BED＝△CEB＝$\frac{1}{2}$△ABC

又 △CEF＝$\frac{1}{2}$△CEB＝$\frac{1}{4}$△ABC

∴ △EFG＝$\frac{1}{2}$△CEF＝$\frac{1}{8}$△ABC

(100) (証) ∵ △ACE＝$\frac{1}{3}$△ABC

△AGE＝△BEF＝△CFG＝$\frac{2}{3}$△ACE＝$\frac{2}{9}$△ABC

∴ △AGE＋△BEF＋△CFG＝$\frac{2}{3}$△ABC

∴ △EFG＝$\frac{1}{3}$△ABC

∵ ∠HBF＝∠KCG　∠HFB＝∠KGC

BF＝CG　∴ △HBF＝△KCG

同理ニテ △HBF＝△LAE＝△KCG＝Aト名ク

∵ △ABF＋△ACE＋△CBG＝△ABC

∴ △ABF＋□FHGC＋□GKLA＋3A＝△ABC

$\therefore\ 3A = \triangle HKL$

$\because\ CF = 2FB$

$\therefore\ \triangle CFH = 2\triangle BFH = 2A = 2\triangle KCG$

$\therefore\ \triangle CBH = 3\triangle KCG \quad \therefore\ BH = 3KG$

$\therefore\ \triangle ABH = 3\triangle AKG$

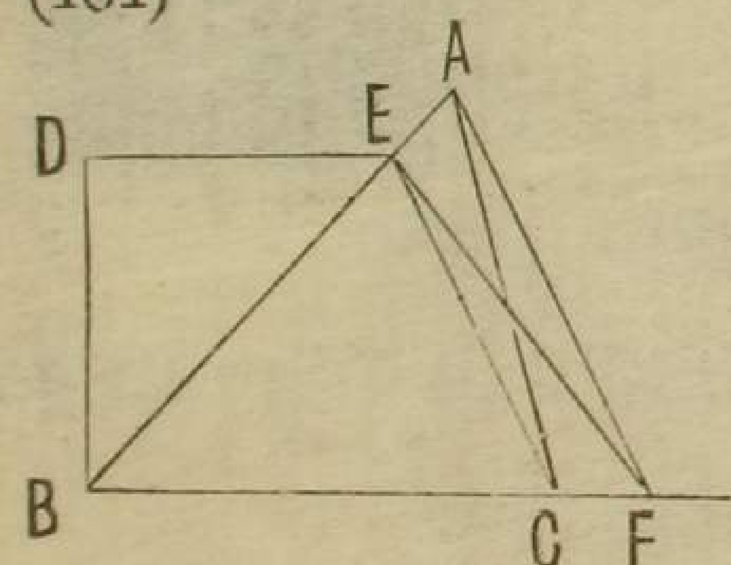
然レモ $\triangle AKG = \triangle CFH = 2A$

$\therefore\ \triangle ABH = 6A = 6\triangle HBF \quad \therefore\ AH = 6FH$

$\because\ \triangle BFH = \frac{1}{7}\triangle ABF = \frac{1}{21}\triangle ABC$

$\therefore\ 3\triangle BFH = 3A = \triangle HKL = \frac{1}{7}\triangle ABC$

(101)



ABC ヲ原三角形トス

(1) (画法)

$BD \perp BC$

BD ＝ 望ム高

$DE \parallel BC$, $AF \parallel EC$

$\triangle BEF$ ハ求ムルモノナリ

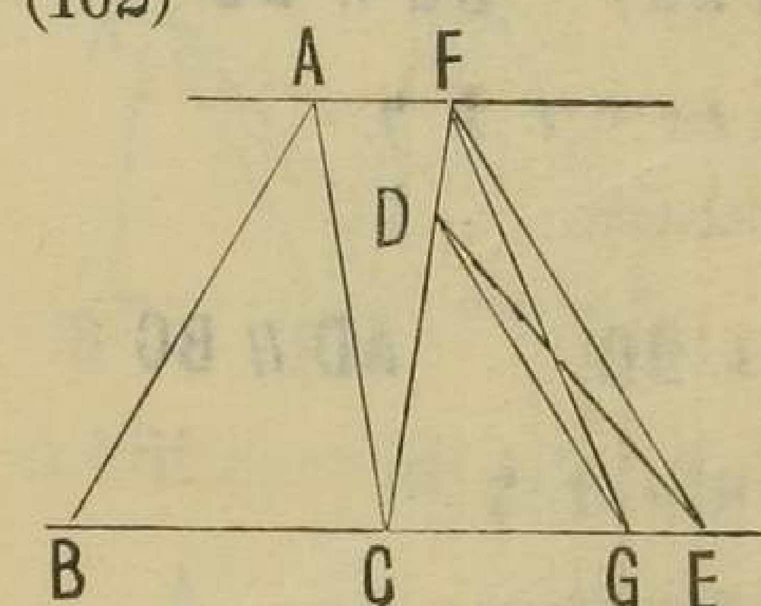
又 DE 線 BA ノ延線ニ會スルモ同法也

(2) BF ＝ 望ム底、$CE \parallel AF$、

$\triangle BEF$ ハ求ムルモノナリ

又 $BF < BC$ ナルモ亦同法ナリ但シ CE ハ BA ノ延線ニ會スベシ

(102)



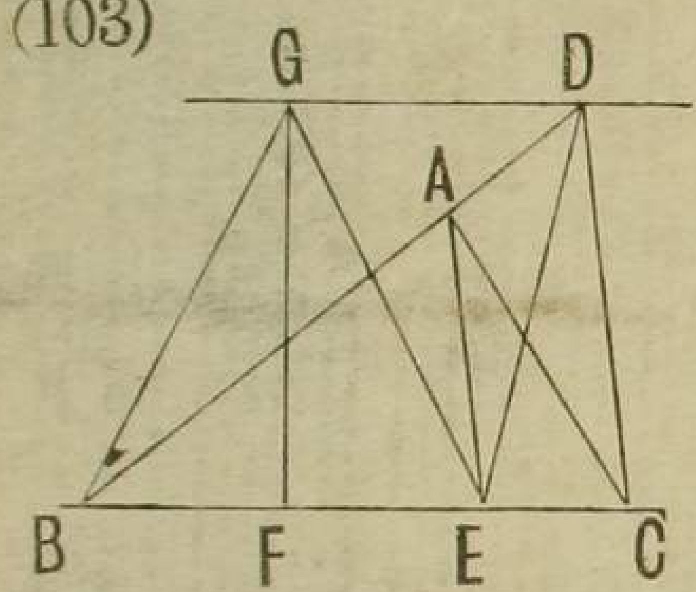
ABC, DCE ヲ二原三角形、BCE ヲ一直線トス

(1) (画法)

$AF \parallel BE$、$DG \parallel FE$、和ト同積ノ三角形ハ BG ヲ底トシ ABC ト同高ノモノナリ (2) 差ト同積ノモノハ BC, CG ノ差

ヲ底トシABCト同高ノモノナリ

(103)



ABCヲ原三角形トス

(画法)

BE＝望ム底長、

$\triangle BDE = \triangle ABC$ ----(101)

$BF = FE$, $FG \perp BC$, $DG \parallel BC$

$\triangle BGE$ハ求ムルモノナリ

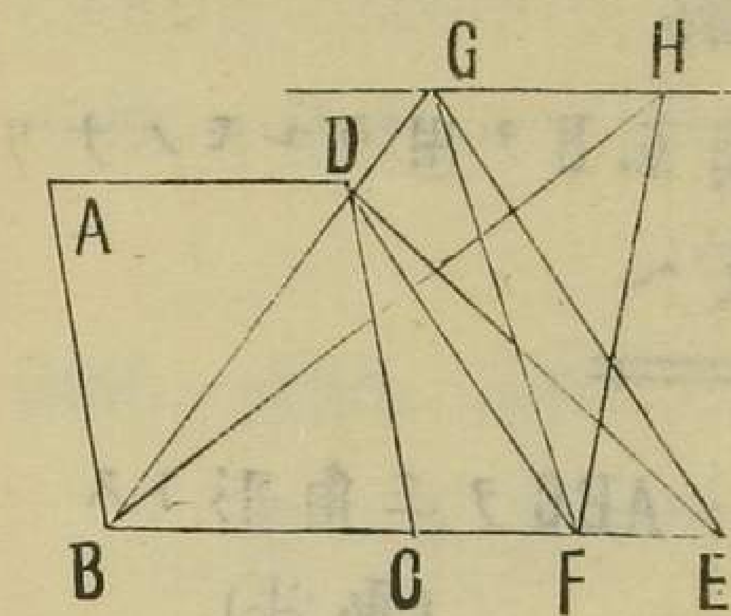
(104) (画法) $BD \perp BC$ $AD \parallel BC$

$\triangle BDC$ハ求ムルモノナリ

(105) ABCDヲ平行形トス

(画法)

$CE = CB$ BF＝望ム一邊



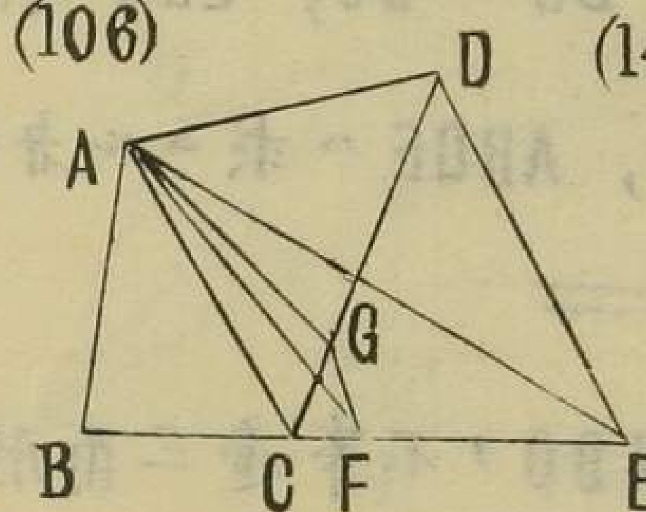
$\triangle BGF = \triangle BDE$

$GH \parallel BE$

$\angle FBH$＝望ム一角、

$\triangle BHF$ハ求ムル者也、

(106)

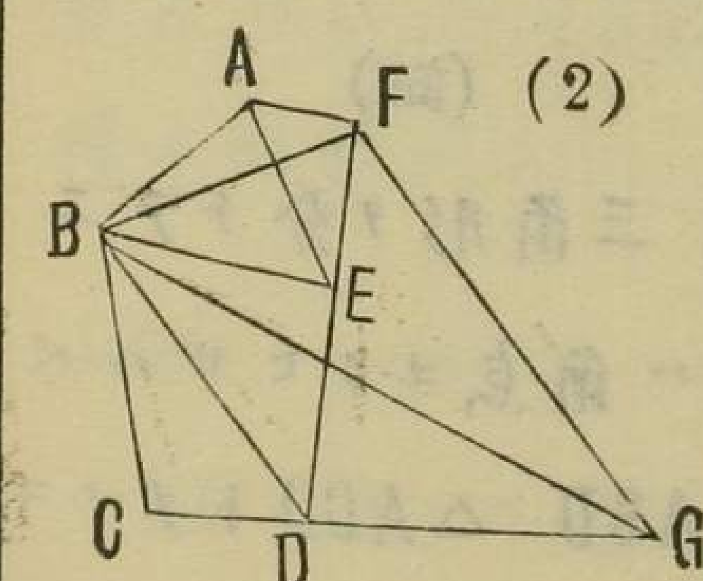


(144) (1) 先ツ四邊形ABCDヲ以テ論ゼン

(画法)

$\triangle ACE = \triangle ACD$

$\triangle ABE$ハ一邊ABト角点Aヲ用ウルモノナリ



(2) 次ニ五邊形ABCDEヲ以テ論ゼン

(画法)

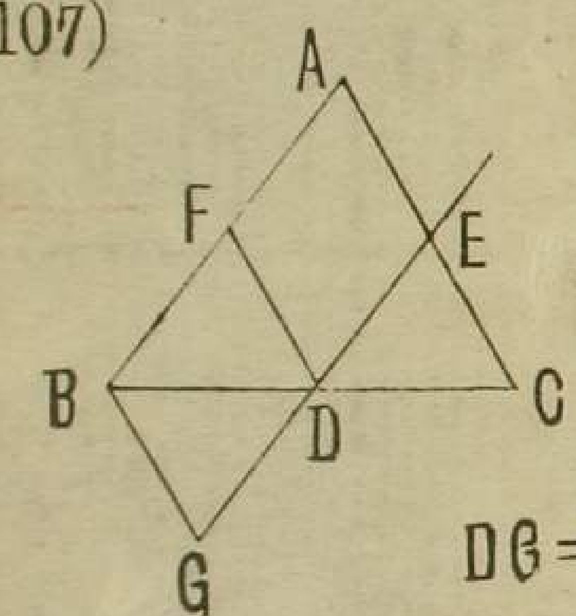
$\triangle BEF = \triangle ABE$

△BDG ＝ △BDF

△BCG ハ一邊 BC ト角点 B ヲ用ウレモノナリ

餘皆之レニ倣ヘ

(107)

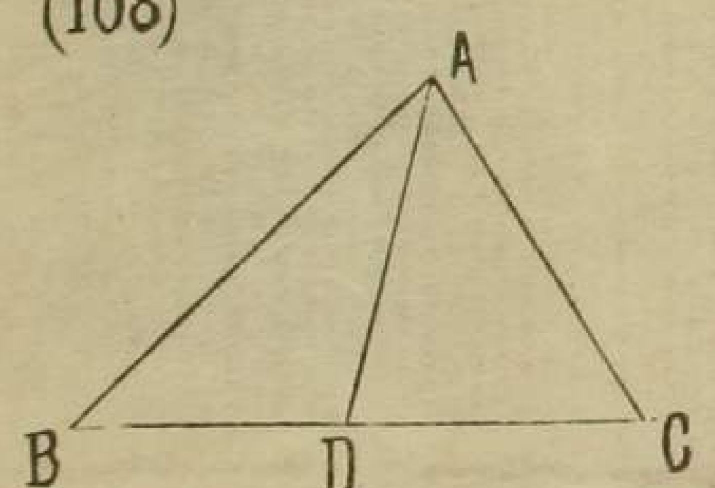


ABC ヲ三角形トス

(画法)

BD ＝ DC, EDG // AB, DG ＝ DE, ABGE ハ求ムル者也

(108)



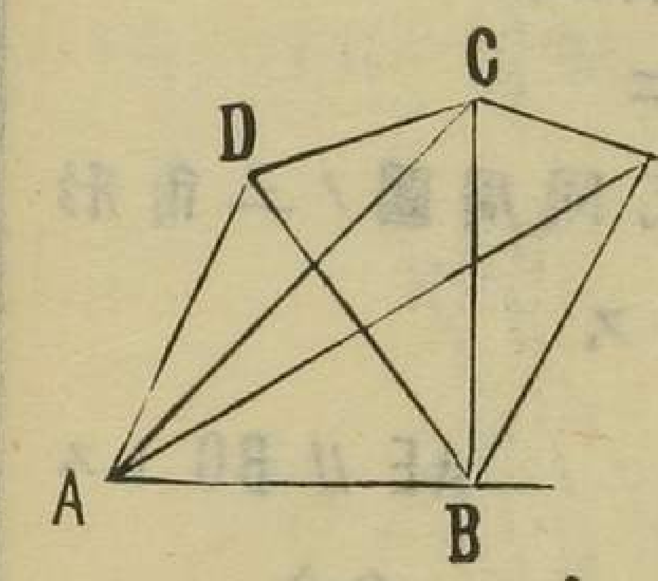
ABC ヲ不等邊三角形、BC ＞ AB ＞ AC トス

(証)

三角形ヲ分チテ二個ノ三角形トナサンニハ角点ヨリセザルベカラズ今 AD ヲ引キ △ABD △ACD トナスニ

∠ABD ＜ ∠ACD ナリ、両三角形中一角等カラズ故ニ等形ナラザルコト明ケシ

(109) ABC, ABD ヲ同底 AB ノ同側ニアル直三角形ト斜三角形ニシテ一邊 BC ＝ BD ナルモノトス



(証)

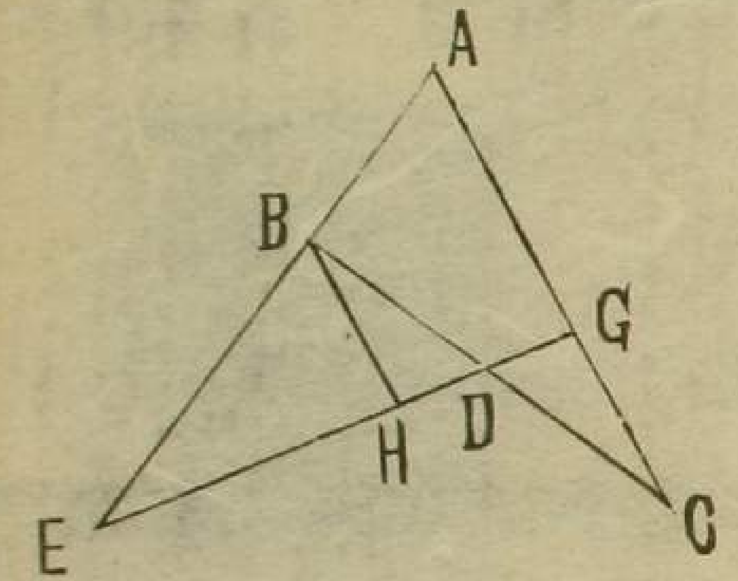
∵ BC ＝ BD

∴ ∠BCD ＝ ∠BDC ＜ r.a.

∴ ∠ABC ＋ ∠BCD ＜ 2r.a.

∴ CD ヲ延セハ AB ノ延線ニ會スベシ ∴ △ABC ノ高サハ最大ナレバ其積モ亦最大ナリ

(110) △ABC ト △AEG ノ底 D ニテ交互シ且ツ BC ハ D ニテ折半サルハモノトス



BH // AC トス

(証)

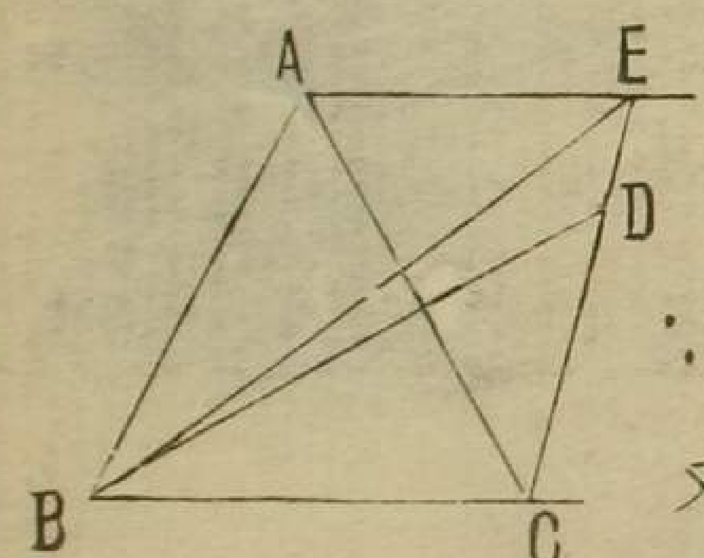
∵ ∠BDH＝∠CDG

∴ ∠DBH＝∠DCG

△BDH＝△CDG

∴ △ABC ＜△AEG

(111)　ABC, BDC ヲ同底同周圍ノ三角形其中チ ABC ヲ二等邊トス



AE // BC トス

(証)

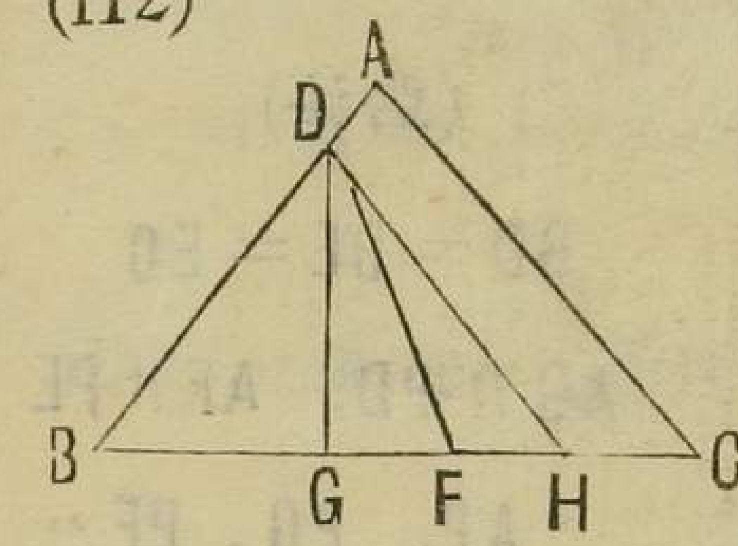
∵ BE＋EC ＞ AB＋AC ---(42)

又　AB＋AC ＝ BD＋DC

∴ D ハ平行線 AE, BC ノ間ニ在リ

∴ △ABC ハ最大ナリ

(112)　ABC ヲ三角形トス



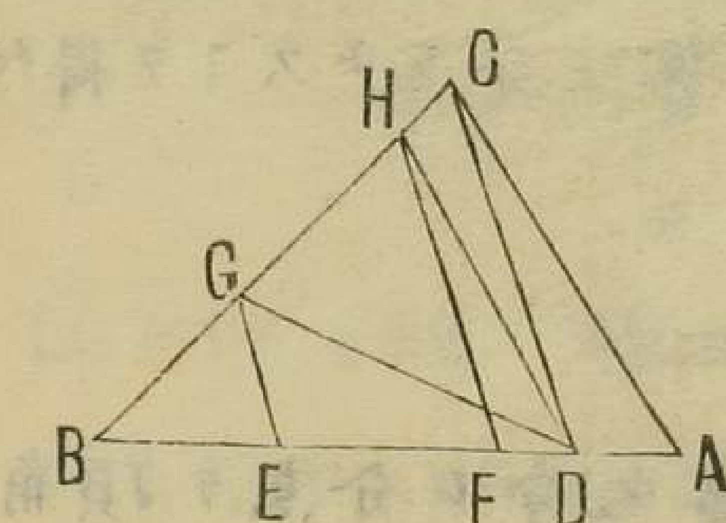
(1)　D ヲ AB 中ノ設点トス

(画法)

DF＝△ABC ノ折半線 --------(6)

FG ＝ FH ＝ $\frac{1}{3}$BF,　DG、DH ハ求ムルモノ也

(又法)

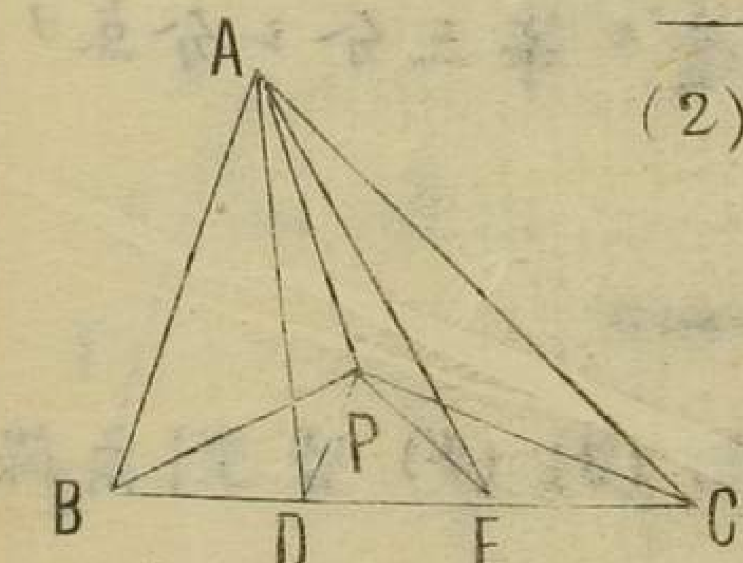


BE ＝ EF ＝ FA

FH // EG // DC

DG, DH ハ求ムル者也

(2)　(画法)

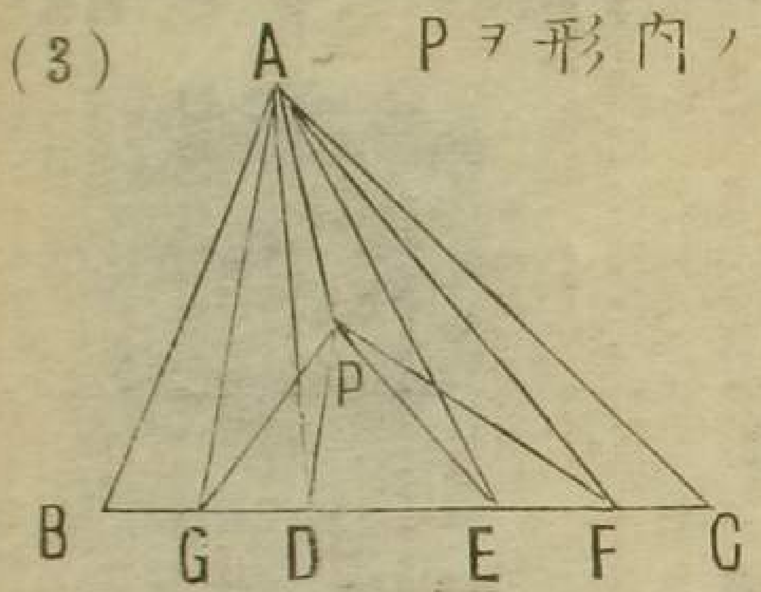


BD ＝ DE ＝ EC

EP // AC,　DP // AB

AP, BP, CP ハ求ムル者也

(3) Pヲ形内ノ一点トス



(画法)

BD＝DE＝EC

AG $\parallel$ PD, AF $\parallel$ PE

AP, PG, PF ハ求ムルモノナリ

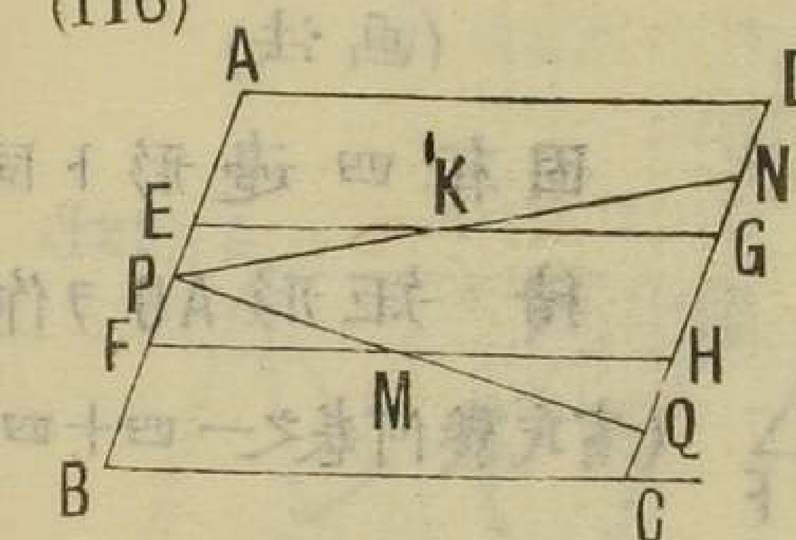
〔答〕 AC（或ハAB）ヲ等分三個トシ上ト同法ヲ以テ△ABCヲ等三分トナスヿヲ得ベシ故ニ以上三法アリ

(113) (画法) 底ヲ等九分シ分点ヲ頂角点ニ繫キ、或ハ三邊ヲ等三分シ分点ヲ相繫グベシ

(114) (画法) (1) (2) (3) (4) 皆對角線ノ交互点ヲ貫キ一線ヲ引キ本形ヲ二分スベシ此線即折半線ナリ

(115) (画法) (114)ト同法ナリ

(116)



ABCDヲ平行形、PヲAB中ノ一点トス

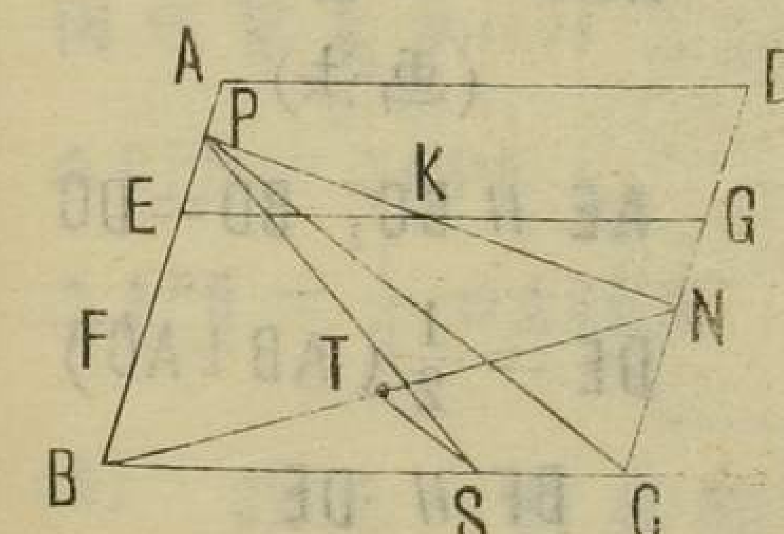
(1) (画法)

AE＝EF＝FB

EG $\parallel$ FH $\parallel$ AD, EK＝KG, FM＝MH

(P点EF中ニアル時) ハ PKN, PMQ ハ求ムル者也

(2)



(P点EF外ニアル時) ハ

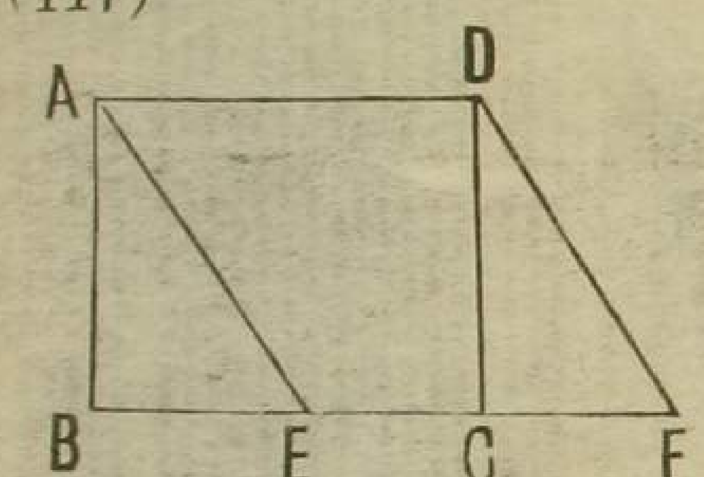
BT＝TN, TS $\parallel$ PC

PKN、PS ハ求ムル者也

(3)　P点、角点（假令ハA）ニアルトキハ(2)ト同法ナリ

(4)　P点E（或ハF）点ト合スルトキハ画法明白ナリ

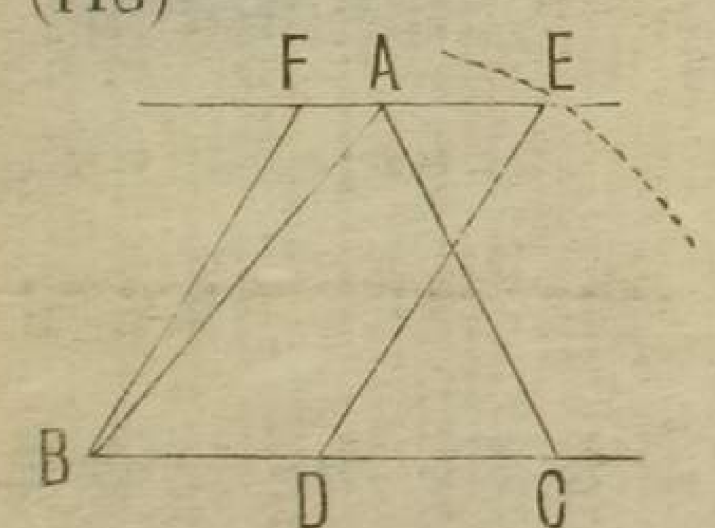
(117)　（画法）



固有四邊形ト同積ノ矩形ACヲ作ル（宥氏幾何巻之一四十四）

AE＝AD,　DF∥AE,　AEFDハ求ムル者也

(118)　ABCヲ三角形トス（画法）



AE∥BC,　BD＝DC

$DE = \frac{1}{2}(AB+AC)$

BF∥DE

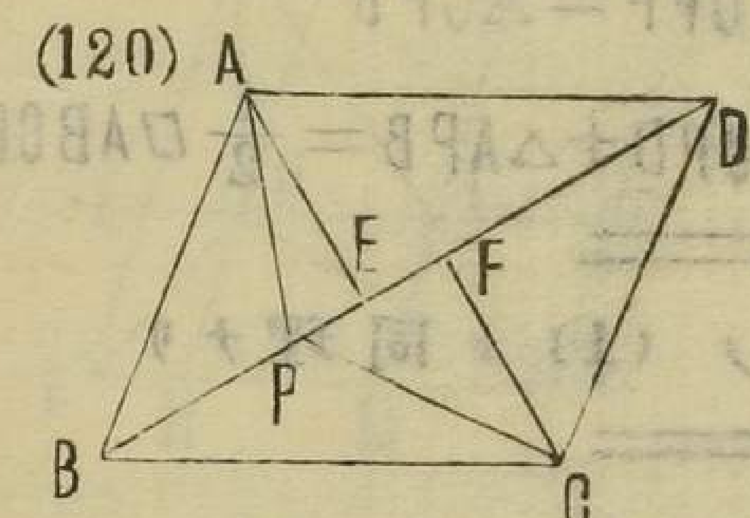
BDEFハ求ムルモノナリ

(119)　ABCDヲ正方形トス

（画法）　ABヲ延シAE＝AC、EF∥AD ACヲ延シEF＝Fニ會セシム

△AEFハ求ムルモノ也

(120)



ABCDヲ平行形トス

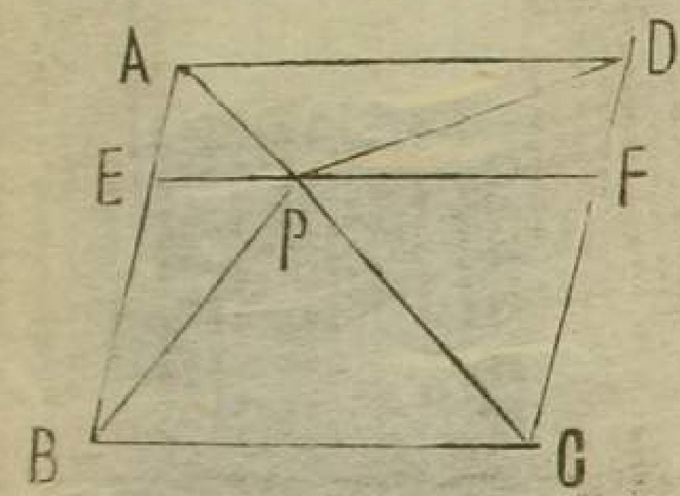
(1)　設点P對角線BD中ニ在ルトキ AE, CF⊥BDトス

（証）　∵ △ABE ≅ △CDF　∴ AE＝CF

同底等高ナレバ △APD＝△CPD

△CPB＝△APB　∴ △APD＋△CPB＝△CPD＋△APB＝$\frac{1}{2}$ □ABCD

(2)　設点P對角線中ニアラザルトキ



EPF $\parallel$ AD トス

(証)

$\because$ AD $\parallel$ EPF

EP + PF = AD

$\therefore \triangle APE + \triangle DPF = \triangle APD$

同理ニテ $\triangle BPE + \triangle CPF = \triangle CPB$

$\therefore \triangle APD + \triangle CPB = \triangle CPD + \triangle APB = \frac{1}{2} \square ABCD$

(121) (証) (120) ノ (1) ト同理ナリ

(122) (証) $\because$ BFC $\parallel$ AD BF + FC = AD

$\therefore \triangle ABF + \triangle CDF = \triangle ADF = \frac{1}{2} \square ABCD$

又 $\because$ AG $\parallel$ DC $\therefore \triangle CDG = \frac{1}{2} \square ABCD$

$\therefore \triangle ABF + \triangle CDF = \triangle CDG$

$\triangle$ CDF ヲ去レバ $\triangle ABF = \triangle CFG$

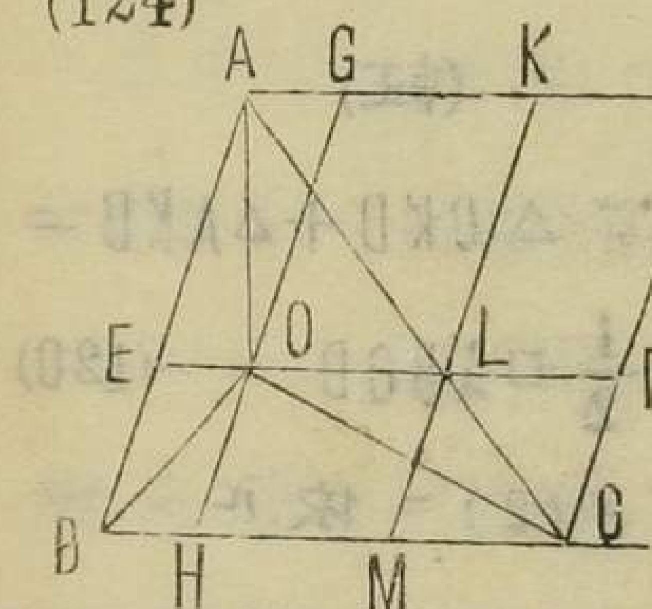
(123) (証) $\because \triangle BKC = \frac{1}{2} \square ABCD$ ---- (122)

對角線ハ相折半スルモノナレバ

BE = DE, $\therefore \triangle BEC = \frac{1}{2} \triangle BDC = \frac{1}{4} \square ABCD$

$\therefore$ ◁BKCE $= \frac{1}{4} \square ABCD$

(124)



EOLF $\parallel$ AD

GOH $\parallel$ AB ナリ

今 KLM $\parallel$ AB トス

(証)

$\because \square DL = \square LB$

$\therefore \square DO = \square LB + \square KO = \square BO + \square HL + \square KO$

$\therefore \square DO - \square BO = \square HL + \square KO$

又 $\because$ BC $\parallel$ OL $\parallel$ AD

$\therefore \square HL = 2\triangle LOC \quad \square KO = 2\triangle LOA$

$\therefore \square DO - \square BO = 2(\triangle LOC + \triangle LOA) = 2\triangle AOC$

(125) (証) ∵ $\triangle CPD = \triangle COD +$ ▱$CODP =$ $\triangle AOB +$ ▱$CODP$ ∴ $\triangle CPD - \triangle APB = \triangle AOB$ $- \triangle APB +$ ▱$CODP =$ ▱$APBO +$ ▱$CODP =$ $\triangle APC + \triangle BPD$

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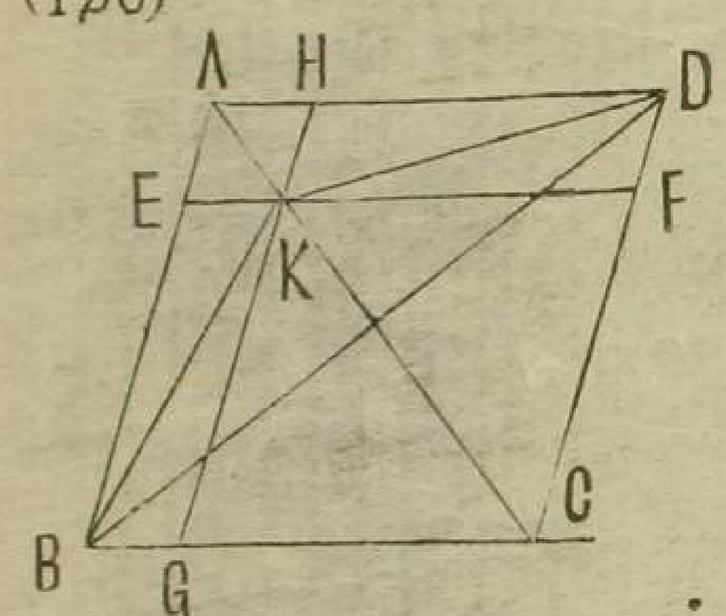
(126)



(証)

∵ $\triangle CKD + \triangle AKB =$ $\frac{1}{2}$ □$ABCD$ ……(120) ノ(2)ニ依ル

∴ $\triangle CKD + \triangle AKB = \triangle ABD$

又 $\triangle DKF + \triangle AKB = \triangle DKH + \triangle AKB$ ヲ去レバ

$\triangle CKF = \triangle AHK + \triangle BKD$

∴ $2\triangle CKF - 2\triangle AHK = 2\triangle BKD$

∴ □$CFKG$ − □$AHKE = 2\triangle BKD$

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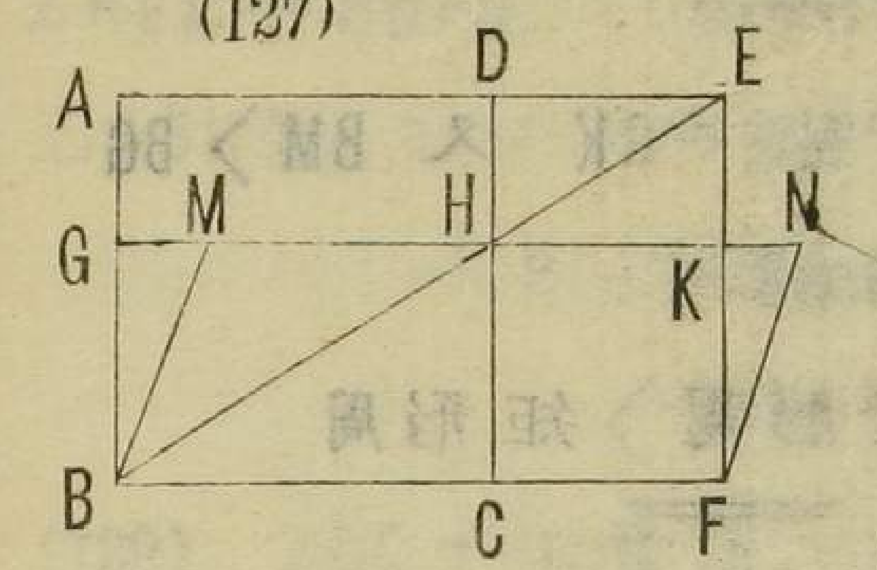
(127)



ABCD ヲ正方形、GBFK ヲ同積ノ矩形トス

(証)

(1) 矩形ノ周圍ハ正方形ノ周圍ヨリ大ナルヲ論ス

□HCFK = □AGHD ニレバ BH, AD, FK ヲ延セバ一点 E ニ會シ BHE ハ平行形 ABFE ノ對角線ナルコ明カナリ

∵ $BF > BC = AB = EF$ ∴ $\angle BEF > \angle EBF$

∴ $\angle HEK > \angle EHK$ ∴ $HK > EK = DH$

∴ $GH + HK + KF > AD + DH + HC$

∴ $GK + KF > AD + DC$

∴ 矩形周 > 正方形周

---

(2) BFNM ヲ矩形 GBFK ト同積ノ平行形ト

シ其周＞矩形周ヲ論ス

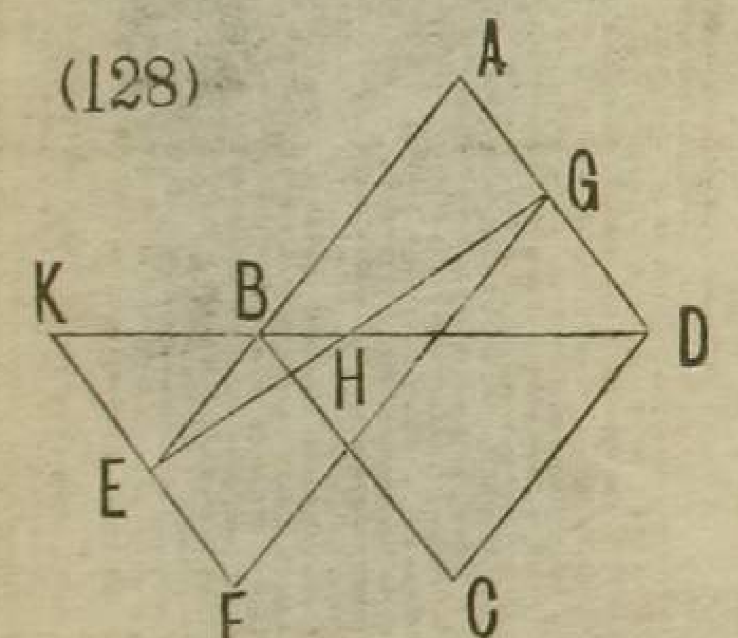
∵ GM＝KN　∴ MN＝GK　又 BM＞BG

∴ MN＋BM＞GK＋GB

∴ 平行形周＞矩形周

(1)(2)ヲ類推セバ正方形ノ周ハ同積ノ他ノ平行形ノ周ヨリ小ナルヲ知ラン

(128)



ABCD, AEFGヲ同角同周圍ノ平行形、其中ABCDヲ等邊ノ者トス

(証)

∵ AE＋AG＝AB＋AD　∴ BE＝GD

∵ FEK∥AD　∴ ∠BKE＝∠ADB＝∠ABD＝∠KBE　∴ KE＝BE＝GD　又 ∠HKE＝∠HDG,

∠HEK＝∠HGD　∴ △HKE＝△HDG＞△HBE、□ABHGヲ加フレバ △ABD＞△AEG

∴ □ABCD＞□AEEG

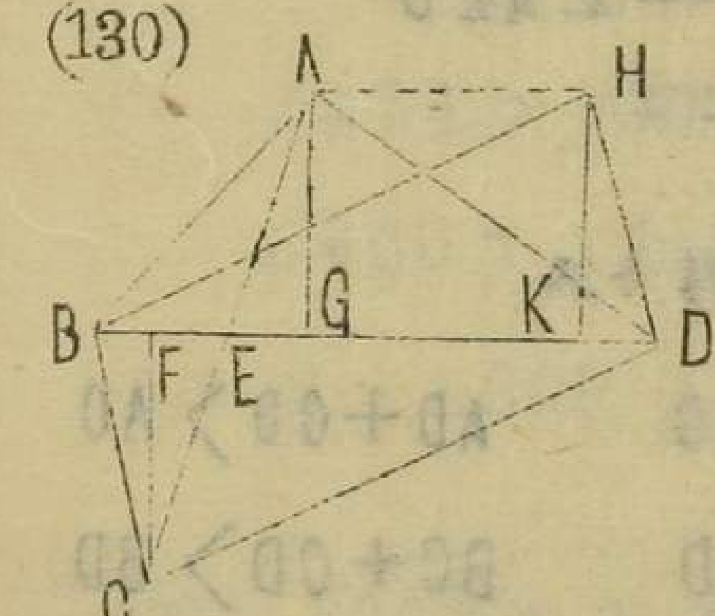
(129) ABCヲ二等邊三角形、AEヲ底BCノ垂線、AECGヲ同積ノ長方形トス

(証) ∵ △ABE≌△ACE, ∴ BE＝EC＝AG

∵ AB＝AC＞AE＝GC

∴ AB＋AC＋BC＞AE＋GC＋EC＋AG

(130)



ABCDヲ四邊形、BDヲ折半線、DH∥＝BC, AG, CF, HK⊥BDトス

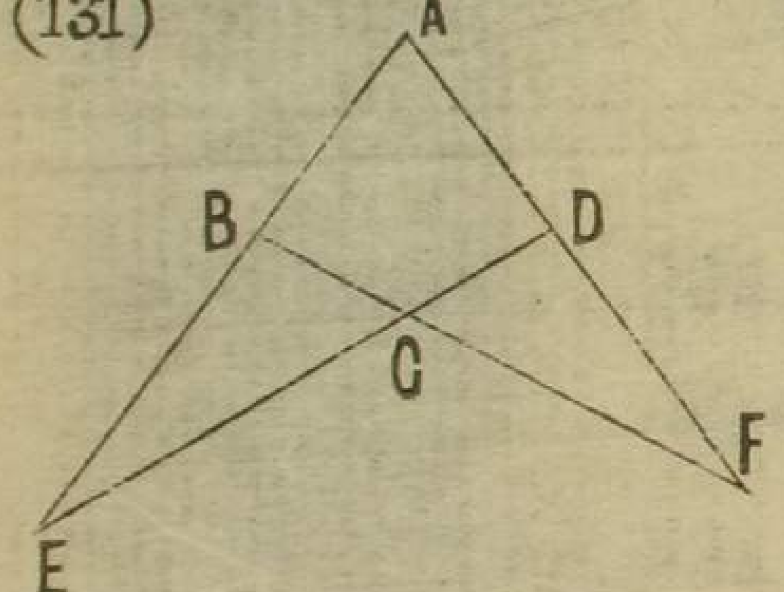
(証) $\because \triangle BHD = \triangle BCD = \triangle ABD$

$\therefore AH \parallel BD \quad \therefore AG = HK$

$\because \triangle BCF \cong \triangle DHK \quad \therefore CF = HK = AG$

$\because \triangle AEG \cong \triangle CEF \quad \therefore AE = EC$

(131) ABCDヲ四邊形、$\angle ABC = \angle ADC$ナリ



(証) $\because \angle ABF = \angle ADE$

$\angle BAF = \angle DAE$

$\therefore \angle AFB = \angle AED$

(132) ABCDヲ四邊形トス

(証) $\because AB + BC > AC \quad AD + CD > AC$

$AB + AD > BD \quad BC + CD > BD$

$\therefore AB + BC + CD + AD > AC + BD$

(133) ABCDヲ不等邊四邊形、Eヲ形内ノ一点トス

(証) $\because AE + EC > AC \quad BE + ED > BD$

$\therefore AE + EB + EC + ED > AC + BD$

(134) ABCDヲ四邊形 ABヲ最大邊、CDヲ最小邊トス

(証) $\because BC > CD \quad \therefore \angle CDB > \angle CBD$

又 $AB > AD \quad \therefore \angle ADB > \angle ABD$

$\therefore \angle ADC > \angle ABC$ 同理ニテ

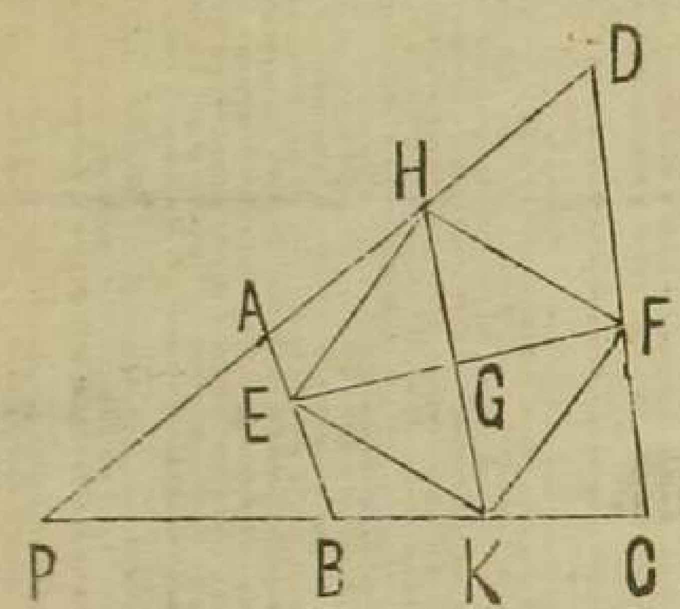
$\angle BCD > \angle BAD$ 相加ヘテ

$\angle ADC + \angle BCD > \angle ABC + \angle BAD$

(135) ABCDヲ不等邊四邊形、

E, F ヲ AB, CD 中ノ点トス

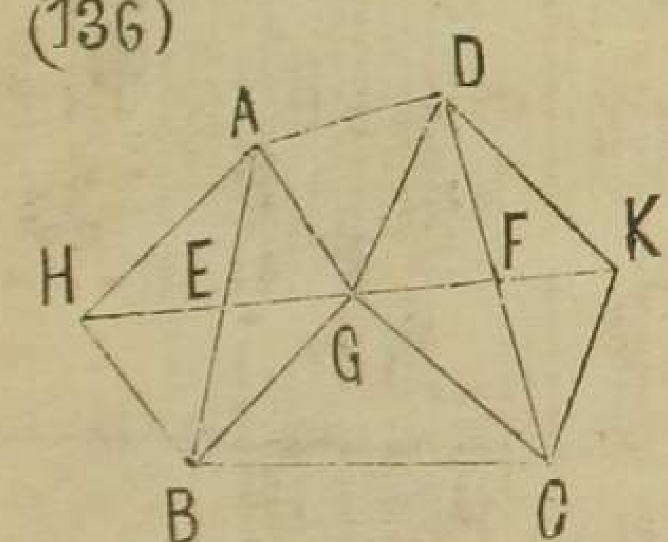
(画法)



EG＝GF、DA, CB ヲ延シ P ニ會セシメ HGK＝2GH ……(3) ニ依ル

HEKF ハ求ムルモノナリ

(136)



ABCD ヲ四邊形トス

AE＝EB, DF＝FC トシ

EG＝GF, GH＝2EG

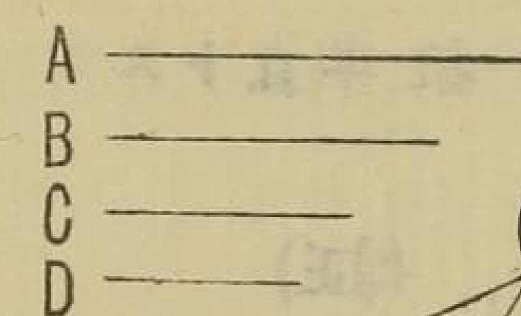
GK＝2GF トスレバ

AHBG, DGCK ハ平行形ナリ今其(証)ヲ下ニ掲グ ∵ AE＝EB　EH＝EG　∠AEH＝∠BEG　∴ AH ≞ BG　∴ AG ≞ BH

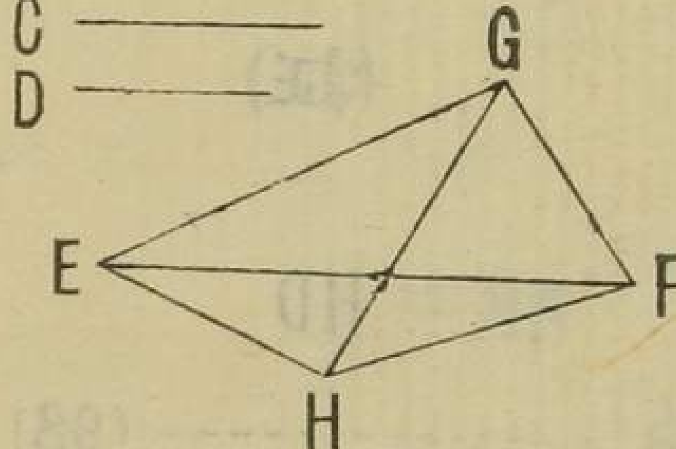
∴ AHBG ハ平行形ナリ

同理同法ヲ以テ DGCK ハ平行形ナル亅明白ナリ

(137)　A, B, C, D ヲ四直線トス



(画法)



EF ヲ四直線中二個ノ和ノ各種ヨリ短キモノトシ

EG＝A　FH＝B

FG＝C　EH＝D　GEHF ハ求ムル者也

〔答〕 四線中ノ三個ノ和他一線ヨリ短キ井能ハザル場合ナリ

(証) ∵ EH＋HF＞EF　EF＋FG＞EG

∴ EH＋HF＋FG＞EG　∴

三線ノ和他一線ヨリ長カラザレバ四邊形ヲ作ス能ハザルナリ

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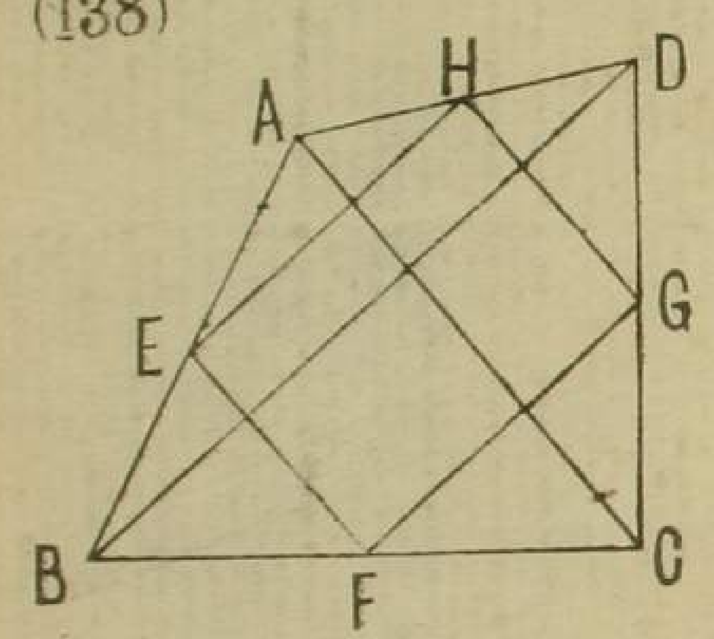
(138)



ABCDヲ四邊形、E, F, G, Hヲ邊ノ折半点トス

(証)

$\because$ AE = EB　又　AH = HD

$\therefore$ EH $\parallel$ BD ---------------- (98)

同理同法ヲ以テ FG $\parallel$ BD,

EF $\parallel$ AC,　HG $\parallel$ AC

$\therefore$ EFGHハ平行形ナリ

$\because$ $\triangle AEH = \frac{1}{4} \triangle ABD$ ---------------- (97)

$\triangle CFG = \frac{1}{4} \triangle CBD$　相加ヘ

$\therefore$ $\triangle AEH + \triangle CFG = \frac{1}{4}$ □ABCD

同理ニテ $\triangle BEF + \triangle DGH = \frac{1}{4}$ □ABCD

$\therefore$ 總和 $= \frac{1}{2}$ □ABCD, 本形ヨリ去リ

□EFGH $= \frac{1}{2}$ □ABCD

---

(139) ABCDヲ不等邊四邊形、相對角点B, DヨリACニ落ル垂線相同トス

(畫法) AE = EC　Eハ求ムルモノナリ

---

(140) ABCDヲ不等邊四邊形、其中ナAB $\parallel$ DC又E, FヲAB DCノ折半点トス

(証) $\because$ AB $\parallel$ DC　AE = EB

$\therefore$ $\triangle AEF = \triangle EBF$　同理ニテ

$\triangle ADF = \triangle BCF$　相加ヘテ

$\therefore$ □AEFD = □EBCF $= \frac{1}{2}$ □ABCD

---

(141) ABCDヲ不等邉四邉形、

Eヲ對角線ノ交互点トス

(証)

今 ∵ $\triangle AED = \triangle BEC$ トスレハ

$\triangle AEB = \triangle AEB$ (+

∴ $\triangle ABD = \triangle ABC$ ABヲ底トシテ同積ナレハ AB ∥ DC

(142) ABCDヲ四邉形、AB ∥ CD、DC ＞ AB AD = BC トス

BE ∥ ADニシテDCニEニ會セシム

ABED＝平行形也

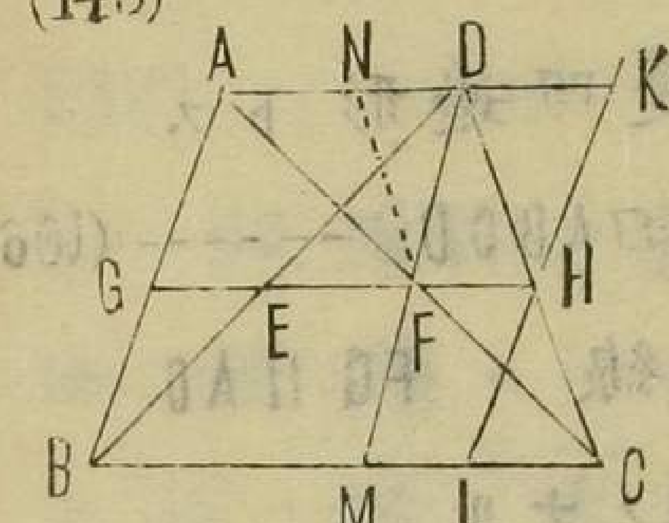
(証) ∵ AD ∥ BE ∴ $\angle ADE = \angle BEC$

又 BE = AD = BC ∴ $\angle BCD = \angle BEC$

又 ∵ $\angle BAD + \angle ADE =$ 2r.a.

∵ $\angle BAD + \angle BCD =$ 2r.a.

(143) ABCDヲ四邉形、AD ∥ BC、E, FヲBD, ACノ折半点トス KHL ∥ ABトス



(証)

∵ AF = FC ∴ DF = FM ---------- (80)

又 DE = EB ∴ EF ∥ BM ---------- (138)

∵ CF = FA FH ∥ AD ∴ CH = HD

此証 FN ∥ HDトシテ得ベシ

∵ $\triangle CHL \cong \triangle DHK$ ∴ CL = DK

∴ $GH = \frac{1}{2}(AK + BL) = \frac{1}{2}(AD + DK + BC - CL) = \frac{1}{2}(AD + BC)$

∵ AD = 2GE = 2FH = GE + FH

∴ $EF = GH - AD = \frac{1}{2}(BC - AD)$

(144)　ABCDヲ不等邊四邊形トス

(画法)　(1) △BAE＝□ABCD -------(106)

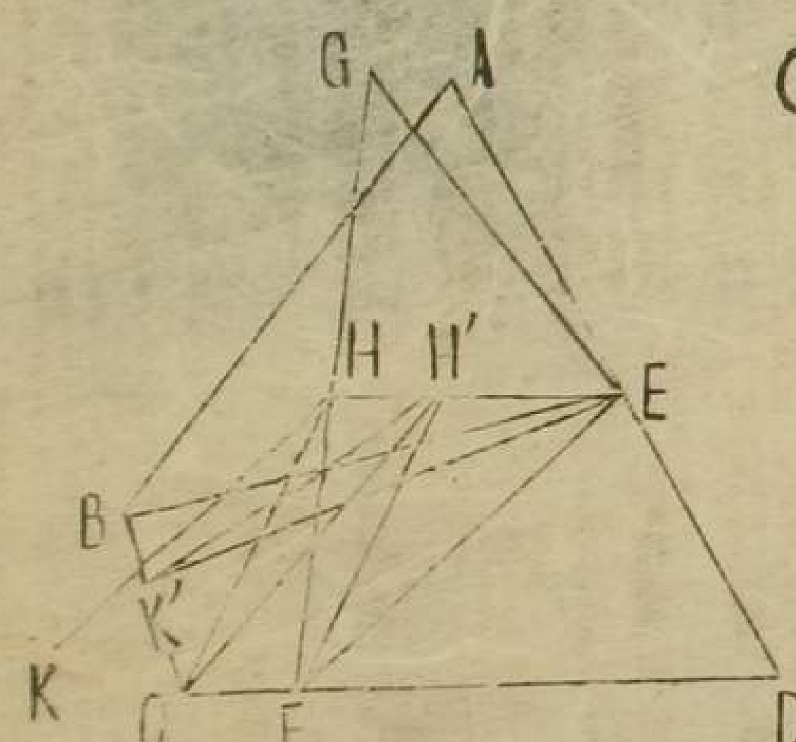
AF＝底BEノ折半線　FG∥AC

AGハ求ムルモノナリ

F点BC中ニ在ラバAFハ即求ムル者也

(又法)　E＝對角線BDノ折半点、

FEG∥他ノ對角線AC　AGハ求ムル者也



(2)　Eヲ一邊AD中ノ一点トス

(画法)

EF＝□BCDEノ折半線、

(上ノ法ニテ)

EF上ニ且ツ△ABEト同側ニ △EEC＝△ABE --------(101)ノ(2)、GH＝HF, HK∥EF、

若シHK線CDニ會セバ此會点トEヲ繋グ線ハ問ニ應ズルモノナリ

今HKハCDノ隣邊BCヲ切レリ然ルトキハ下条ノ如クセヨ

FH′∥CH, H′K′∥EC、EK′ハ求ムル者也

若HK線BCヲモ切ラズシテ其隣邊ABヲ切ラバ CH″∥BH′, H″K″∥EB トス 即 EK″ハ求ムルモノナリ

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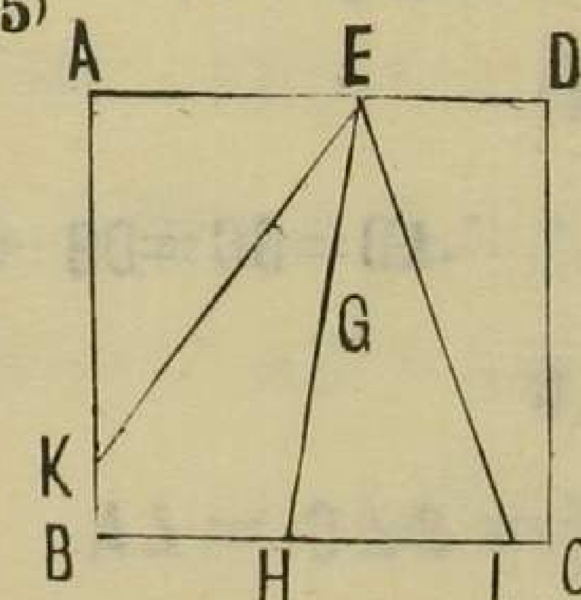
(145)



ABCDヲ正方形、

EヲAD中ノ一点、

Gヲ對角線ノ交互点トス

(画法) $EK = \square ABHE$ ノ折半線、

$EL = \square EHCD$ ノ折半線 ‥‥‥‥‥(144)

EK、EL ハ求ムルモノナリ

---

(146) 一對角線他ノ對角線ヲ折半スルトキハ第一對角線ノ正中点ヨリ角点ニ繋ゲバ本形ヲ四等分ス‥‥‥‥(130)

餘ノ不等邊四邊形ニ至リテハ能ハザルモノナリ其(証)明カナレバ爰ニ説カズ

---

(147) ABC ヲ直三角形、

$\angle ABC$ ヲ直角、$\angle A = 2\angle C$ トシ、

D ヲ AC ノ折半点トス

(証) $\because \angle ABC = r.a. \quad \therefore AD = DC = DB$ ‥(32)

$\because \angle CBD = \angle C$

又 $\angle ADB = \angle CBD + \angle C = 2\angle C = \angle A$

$\therefore AB = BD = AD = DC$

$\because BC^2 = AC^2 - AB^2 \qquad AC^2 = 4AB^2$

$\therefore BC^2 = 3AB^2$

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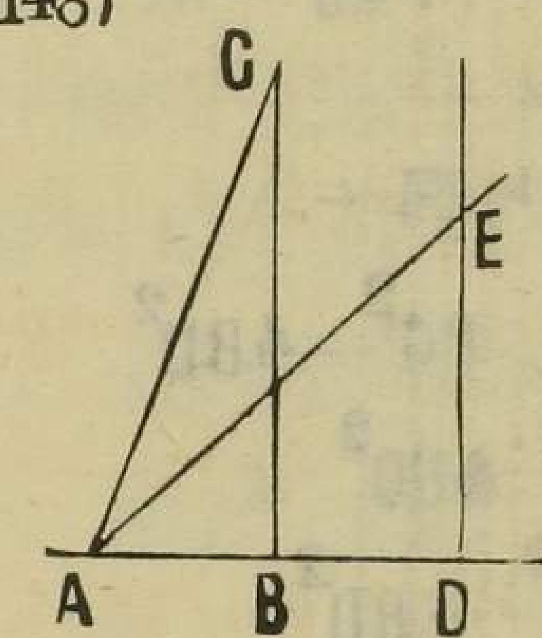
(148)



AB ヲ固有線トス

(画法)

$AD = 2AB$

$DE \perp AD \perp BC$

$DE = AD$

$AC = AE$ ABC ハ求ムルモノナリ

---

(149) ABC ヲ三角形、AD ヲ底 BC（或ハ其延線）ニ下セル垂線トス

(証) $\because AD^2 = AB^2 - BD^2 = AC^2 - CD^2$

$\therefore AB^2 - AC^2 = BD^2 - CD^2$

---

(150) ABC ヲ直三角形、$\angle B$ ヲ直角、D ヲ BC ノ正中点、$DE \perp AC$ トス

(証) $\because AD^2 = AE^2 + DE^2 \quad BD^2 = CD^2 = CE^2 + DE^2$

$\therefore AD^2 - BD^2 = AE^2 - CE^2 \quad \therefore AB^2 = AE^2 - CE^2$

---

(151) (作圖) ハ (150) ト同シ

(証) $\because AB^2 = AC^2 - BC^2 \quad BC^2 = 4BD^2$

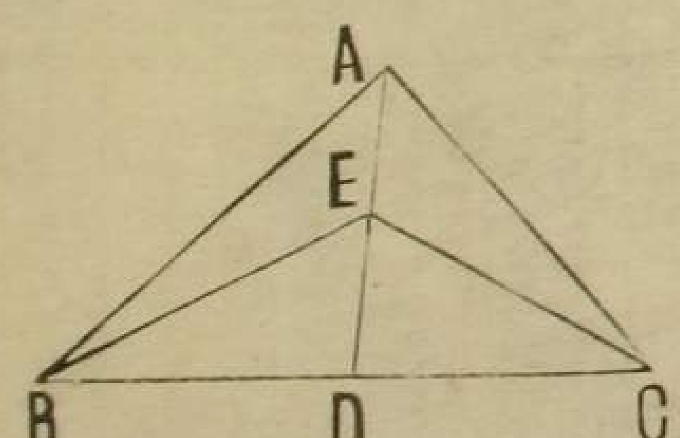
$\therefore AB^2 = AC^2 - 4BD^2$

$\therefore AB^2 + BD^2 = AD^2 = AC^2 - 3BD^2$

---

(152) ABC ヲ三角形 AD ヲ底 BC ノ折半線トス

(証) 若 $\angle BAC$ = r.a. ナラズトセバ



$AD = BD = DC$

ナラズ......(32)

今 $AD > BD = DC$ ト

定ムレバ $ED = BD = DC$ ナリ

$\therefore BC^2 = BE^2 + EC^2$ ---------- (32) ニテ

又 $BC^2 = 4ED^2 \quad \therefore 8ED^2 = BC^2 + BE^2 + EC^2$

又 $8AD^2 = AB^2 + BC^2 + AC^2$ -------- (本文)

$\therefore 8ED^2 - (BE^2 + EC^2) = 8AD^2 - (AB^2 + AC^2)$

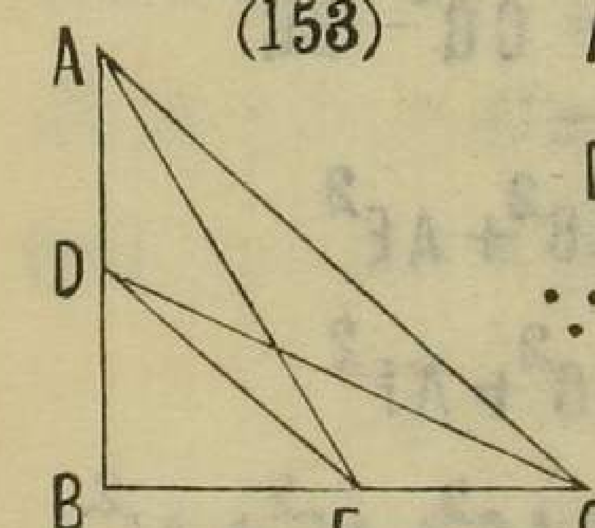
而メ $AB > BE \quad AC > EC$ ナル丁明カナリ

$\therefore AB^2 + AC^2 > BE^2 + EC^2$

$\therefore 8ED^2 > 8AD^2 \quad \therefore ED > AD$

小線大線ヨリ大ナルノ理ナシ故ニ

$\angle BAC$ 直角ナラザルベカラズ

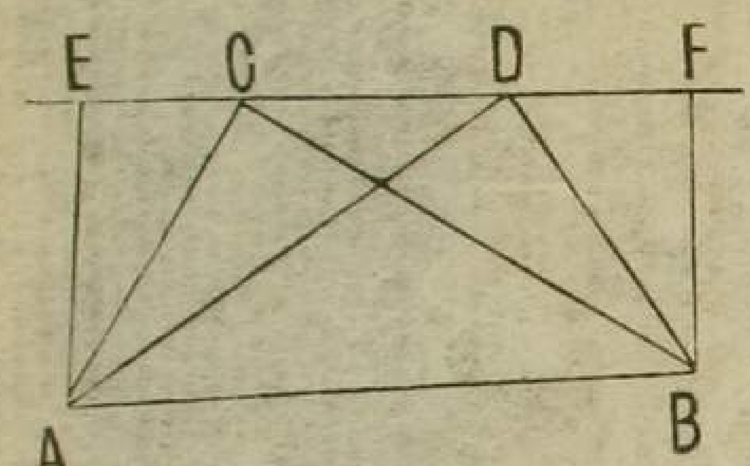


(153) ABC ヲ直三角形、$\angle B$ ヲ直角、$DE \parallel AC$ トス (証)

$\because AE^2 = AB^2 + BE^2,\ CD^2 = BC^2 + BD^2$

$\therefore AE^2 + CD^2 = AB^2 + BC^2 + BE^2 + BD^2 = AC^2 + DE^2$

(154)



(証)

$\because AB^2=AC^2+BC^2$

$AB^2=AD^2+BD^2$

$\therefore AC^2+BC^2=AD^2+BD^2$

$\therefore AE^2+EC^2+CF^2+BF^2=AE^2+DE^2+DF^2+BF^2$

$\therefore EC^2+CF^2=DE^2+DF^2$

(155) (証) $\because AC^2=CD^2+AD^2$

又 $AB^2=BD^2+AD^2$ $\therefore AC^2-AB^2=CD^2-BD^2$

同理ニテ $CG^2-BG^2=CD^2-BD^2$

$\therefore AC^2-AB^2=CG^2-BG^2$

(156) (証) $\because BE^2=AB^2+AE^2$

$CF^2=AC^2+AF^2$

$\therefore 4(BE^2+CF^2)=4(AB^2+AC^2+AE^2+AF^2)$

又 $AB^2+AC^2=BC^2$ $4AE^2=AC^2$

$4AF^2=AB^2$

$\therefore 4(BE^2+CF^2)=5(AC^2+AB^2)=5BC^2$

(157) AC ハ底ナリ

(証) $\because AB^2=BC^2=BD^2+CD^2$

$AC^2=AD^2+CD^2$

$\therefore AB^2+BC^2+AC^2=AD^2+2BD^2+3CD^2$

(158) ABCD ヲ斜方形、E ヲ對角線 AC, BD ノ交互点トス

ABCD ハ平行形ニシテ對角線ハ相互ニ折半スルノ理ハ別ニ証スル迄モナシ

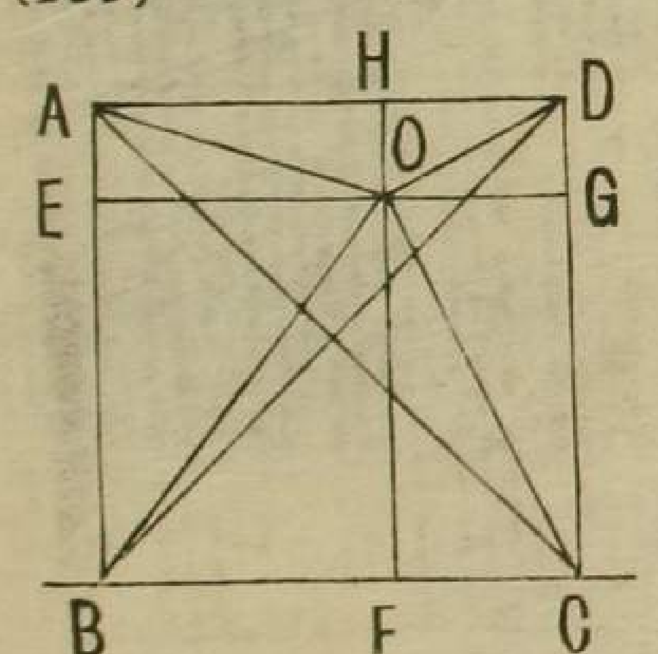
(証) $\because AD=DC$ $AE=CE$

$\angle DAE=\angle DCE$

$\therefore \angle AED=\angle CED=\angle AEB=\angle CEB=$ r∠.

$$\because\ AB^2 = AE^2 + BE^2 = AE^2 + DE^2$$
$$BC^2 = CE^2 + BE^2 = AE^2 + DE^2$$
$$CD^2 = CE^2 + DE^2 = AE^2 + DE^2$$
$$AD^2 = AE^2 + DE^2 \quad (+$$
$$\therefore\ AB^2 + BC^2 + CD^2 + AD^2 = 4AE^2 + 4DE^2$$
$$4AE^2 = AC^2 \qquad 4DE^2 = BD^2$$
$$\therefore\ AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

(159)



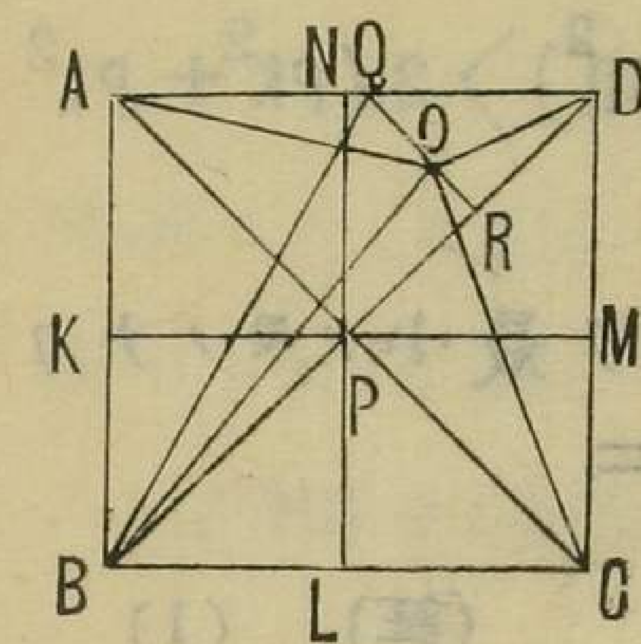
ABCD ヲ正方形、Oヲ其内ノ一点、EOG $\perp$ AB FOH $\perp$ BC トス

(1)　(証)

$$\because\ AO^2 = AE^2 + EO^2 \qquad AE = HO$$
$$\therefore\ AO^2 = HO^2 + EO^2$$ 同理ニテ $$BO^2 = FO^2 + EO^2$$

又 $CO^2 = FO^2 + GO^2 \qquad DO^2 = HO^2 + GO^2$

$$AO^2 + BO^2 + CO^2 + DO^2 = 2(HO^2 + EO^2 + FO^2 + GO^2)$$



(2)　O点Pニ移ルトキハ

$$AP = BP = CP = DP$$

又 $PM = PN = PK = PL$ 也

QOR $\parallel$ AO トス

$$\because\ \angle APD = \angle QRD = \text{r.a.}$$

又 $\angle PAD = \angle RQD = \angle RDQ \quad \therefore\ RD = RQ$

$$\because\ BQ^2 > AB^2,\quad BQ^2 = QR^2 + RB^2 = RD^2 + RB^2$$
$$AB^2 = AP^2 + BP^2 = DP^2 + BP^2$$
$$\therefore\ RD^2 + RB^2 > DP^2 + BP^2$$
然レ氏 $BO^2 + DO^2 > RB^2 + RD^2$
$$\therefore\ BO^2 + DO^2 > DP^2 + BP^2$$

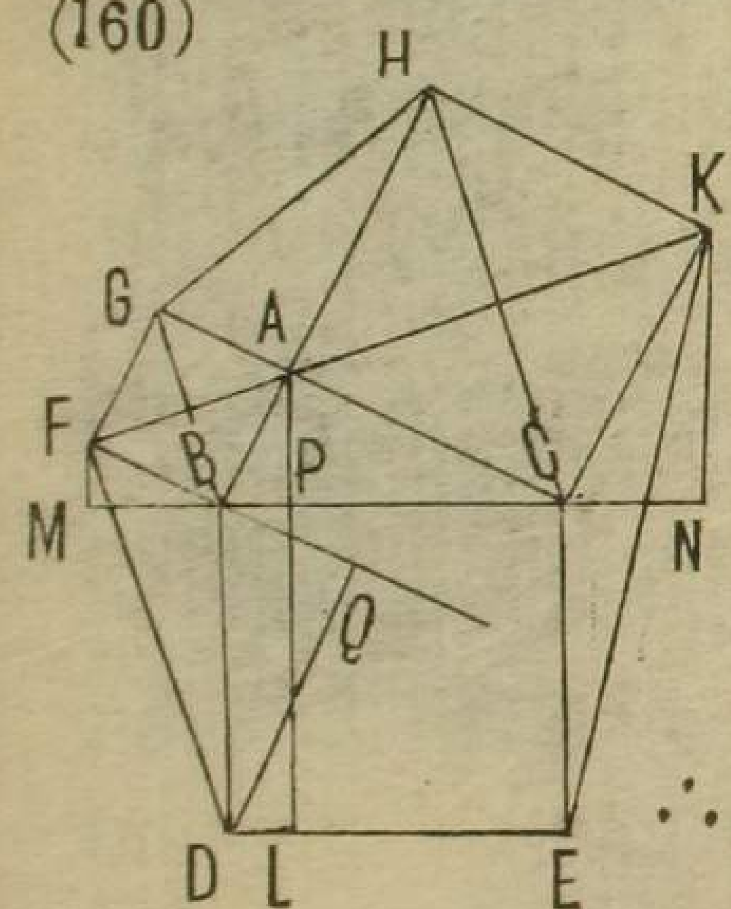
同理ニテ　$AO^2+CO^2>AP^2+CP^2$

∴　$AO^2+BO^2+CO^2+DO^2>AP^2+BP^2+CP^2+DP^2=2(PK^2+PL^2+PM^2+PN^2)$

∴　$2(HO^2+EO^2+FO^2+GO^2)>2(PK^2+PL^2+PM^2+PN^2)$

∴　正方形ノ和両ナガラ最小ノモノナリ

(160)



(証)　(1)

∵ ∠FAB＝∠CAK＝$\frac{1}{2}$ r.a.

∴ ∠FAB＋∠CAK＝r.a.

又 ∠BAC＝r.a.

∴ ∠FAB＋∠BAC＋∠CAK＝2r.a.

∴　FA，AKハ一直線中ニアリ

(2)　∵ ∠ABF＝∠CBD＝r.a.

∴ ∠ABF＋∠CBD＝2r.a.　∴ ∠DBF＋∠ABC＝2r.a.

又 ∠DBF＋∠BDF＋∠BFD＝2r.a.

∴ ∠ABC＝∠BDF＋∠BFD

同理ニテ ∠ACB＝∠CEK＋∠CKE

然レドモ ∠ABC＋∠ACB＝r.a.

∴ ∠BDF＋∠BFD＋∠CEK＋∠CKE＝r.a.

(3)　∵ ∠ABG＝∠AHC 而シテ BAHハ一直線ナレバ　BG // CH

(4)　AL，BCノ交互点ヲPトシBCヲ左右ニ延シ無線FM，KNヲ落ス

∵ ∠ABP＋∠FBM＝r.a.　∴ ∠FBM＝∠BAP

∠BFM＝∠ABP　又 BF＝AB

∴ FM＝BP　BM＝AP

同理ニテ $KN=CP$　　$CN=AP$

$\therefore FM+KN=BP+CP=BC$　又 $BM=CN$

---

(5) 垂線 DQ ヲ FB ノ延線ニ落ス

$\because \angle DBC=\angle ABQ$　$\therefore \angle DBQ=\angle ABC$

又 $\angle BQD=\angle BAC$　　$BD=BC$

$\therefore BQ=BA=BF$　　$DQ=AC$

$\triangle BQD=\triangle BAC$ 又同底同高ナレバ

$\triangle BQD=\triangle BDF$　$\therefore \triangle BDF=\triangle BAC$

同理ニテ　$\triangle CEK=\triangle BAC$

又 $\triangle GAH=\triangle BAC$ タルコト明白ナリ

---

(6) $\because FQ=2BF=2AB$　　$DQ=AC$

$FD^2=FQ^2+DQ^2=4AB^2+AC^2$

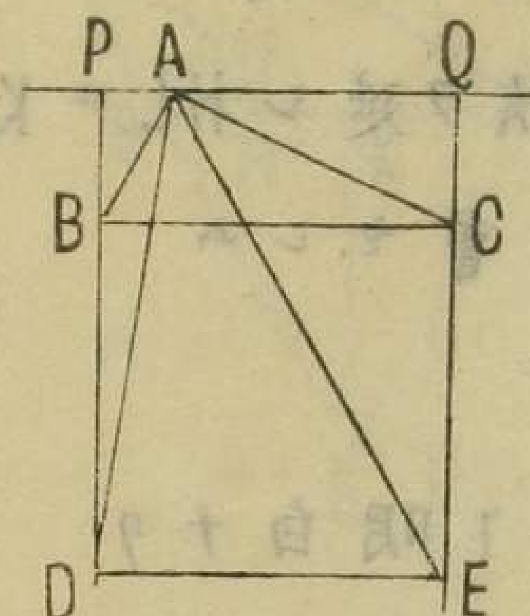
同理ニテ $EK^2=4AC^2+AB^2$ タルコト明白

$GH^2=AG^2+AH^2=AB^2+AC^2$ 相加フレバ

$\therefore FD^2+EK^2+GH^2=6(AB^2+AC^2)=6BC^2$

---

(7)



DB, EC ヲ延シ

$PAQ \parallel BC$ トス

$\because AC^2=AQ^2+CQ^2$

$AB^2=AP^2+BP^2$

又　$BP=CQ$

$\therefore AC^2-AB^2=AQ^2-AP^2$

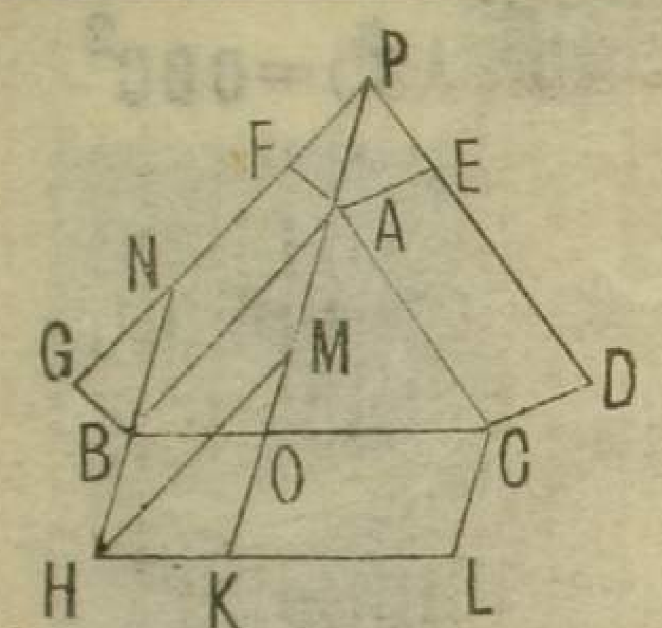
$\because AE^2=AQ^2+EQ^2$　　$AD^2=AP^2+DP^2$

又 $DP=EQ$　$\therefore AE^2-AD^2=AQ^2-AP^2$

$\therefore AC^2-AB^2=AE^2-AD^2$

---

(101) ABC ヲ三角形、AG, AD ヲ随意ノ平行形、P ヲ DE, GF ノ延線ノ交互点、BL ヲ平行形其邊 $BH \stackrel{\parallel}{=} PA$ トス



HBヲ延シ $GP=N$ ニ會セシメ

$HM // BA$、

PAヲ延シ $HL=K$ ニ會セシム

(証) $PA=BN=BH=AM$

又 $NP=BA=HM$ タルコ明白ナリ

同底同高ナルヲ以テ

$\because \square ABHM=\square OBHK,\ \square ABNP=\square ABGF$

又 $\square ABHM=\square ABNP$

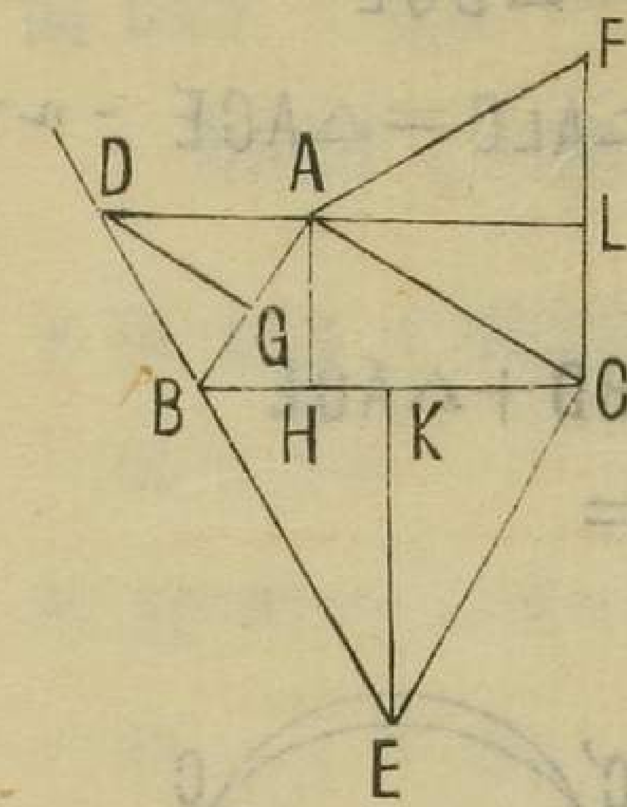
$\therefore \square OBHK=\square ABGF$ 同理ニテ

$\square OCLK=\square ACDE$ (+

$\therefore \square BHLC=\square ABGF+\square ACDE$

(102) BACヲ直三角形、∠Aヲ直角、

$\angle B=\frac{2}{3}$ r.a.



ABD, BCE, ACFヲ等邊三角形トス

$AH\perp BC$,

$DG\perp AB$, $EK\perp BC$,

$AL\perp CF$ トス

(証)

$\because AD=AB,\ \angle DAG=\angle ABH=\frac{2}{3}$ r.a.

$\angle AGD=\angle AHB=$ r.a.

$\therefore \triangle ADG=\triangle ABH$

上ト同理同法ニテ

$\triangle AHC=\triangle ALC$ 又 $\triangle EKC=\triangle ABC$

$\because \triangle ABH+\triangle AHC=\triangle ADG+\triangle ALC$

$\therefore 2\triangle ABC=2\triangle ADG+2\triangle ALC$

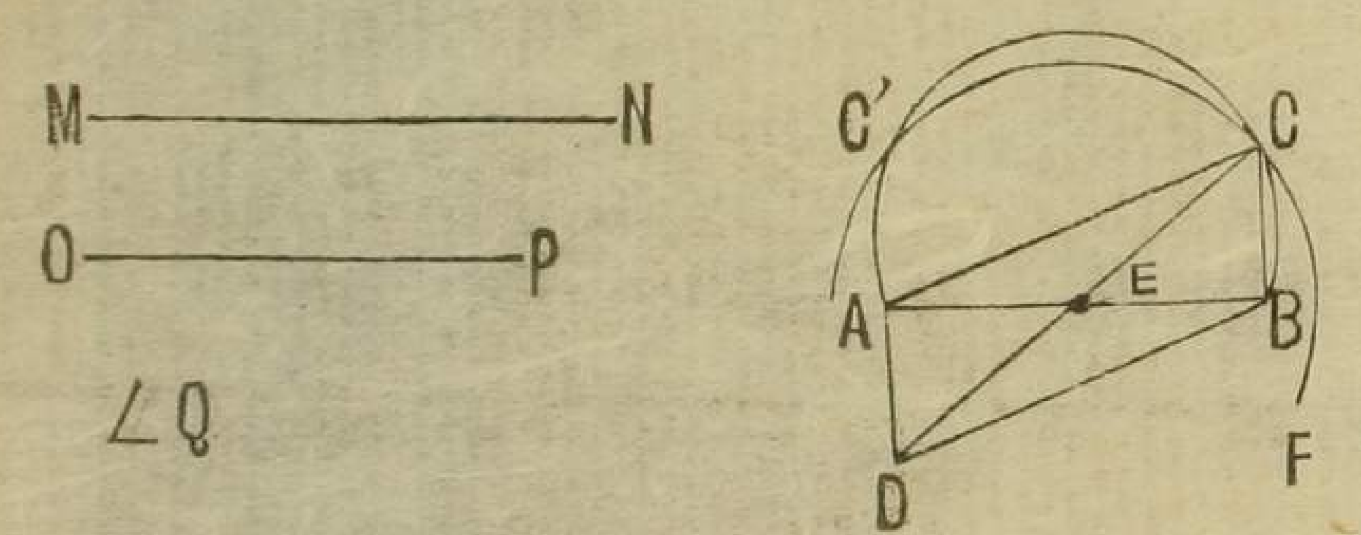
又 2△ABC＝2△EKC＝△BCE

2△ADG＝△ABD　　2△ALC＝△ACE ナルヿ

証スル迄モナシ

∴　△BCE＝△ABD＋△ACE

(85)



MN, OP ヲ二對角線　∠Q ヲ一角トス

(画法)

AB＝OP（∠Q＞ra. ノ時ハ AB＝MN トナスベシ）

AB ノ折半点 E ヲ中心トシ $\frac{1}{2}$MN ヲ半徑トシテ圏 C′CF ヲ画キ又 ⌓ACB＝∠Q ヲ含ム欠圓トシ ----------（巻之三ノ三十三）

圏 C′CF ニ C′,C ニ會セシメ CE（C′E）＝ED トス

ACBD（AC′BD）ハ求ムルモノナリ

欠圓 ACB ガ圏 C′CF ニ觸ルヽ時ヲ以テ ∠Q ノ限リトス

此題巻之三中ノ設題ニ依ラザレバ穏當ノ解ヲ下ス能ハス故ニ特ニ此巻末ニ附シテ以テ識者ノ高論ヲ仰グ

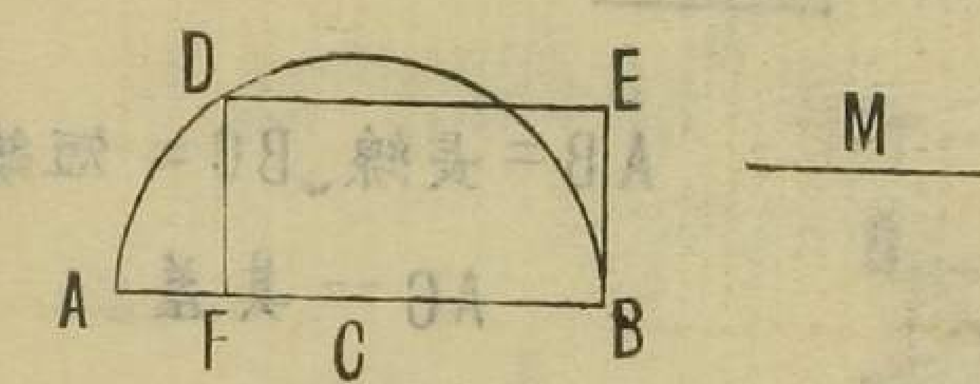
幾何問題解

巻之二

中川將行　荒川重平

同著

(1)



ABヲ二分スベキ一線Mヲ固有正方形ノ一邊トス

(画法)

C＝ABノ折半点

CA＝BC＝ADB半圓ノ半徑

C＝半圓ADBノ中心

$BE = M$ 且ッ $\perp AB$

$ED \parallel BA$　　$DF \parallel EB$

然カスレバ F ハ所求ノ點ナリ

上ノ画法ニ於ED線半圓ニ切觸スルトキハ正方形ハ最大ナリ即チ M 線 $= \frac{1}{2}AB$ ナルトキヲ云フ

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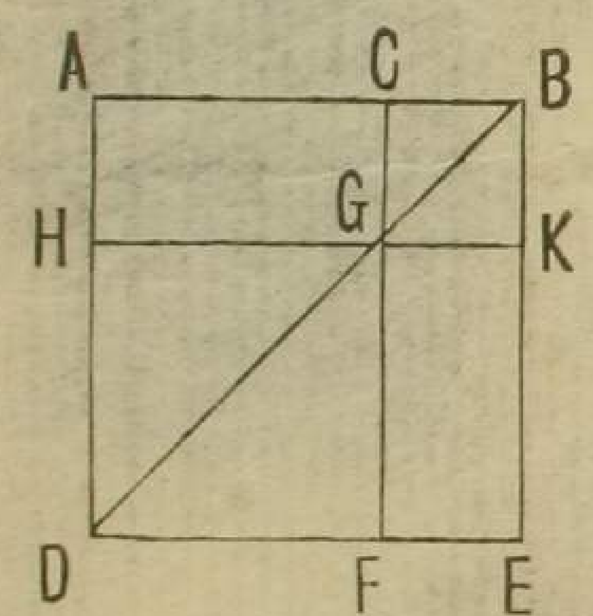
(2)



AB = 長線、BC = 短線

AC = 其差

(画法)

$ABED$ = AB 上ノ正方形

$CF \parallel BE$

$G = BD$ ト CF ノ交点

$HGK \parallel BA$ 若ハ DE

$\because \square AG = \square GE$　　$\therefore \square AK = \square CE$

$\therefore \square AK + \square CE = 2\square AK$

然ルニ $\square AK + \square CE = \ulcorner AKF + \square CK$

且ッ $\square AK = AB.BC$

$\therefore \ulcorner AKF + \square CK = 2AB.BC$

然ルニ $\square AE + \square CK = AB^2 + BC^2$

$\therefore \square AE - \ulcorner AKF = AB^2 + BC^2 - 2AB.BC$

然ルニ $\square AE - \ulcorner AKF = \square HF = AC^2$

$\therefore AC^2 = AB^2 + BC^2 - 2AB.BC$

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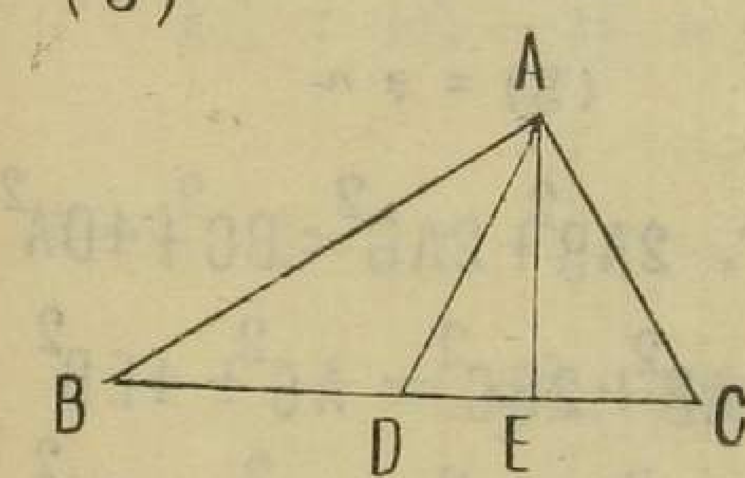
(3)



ABC ハ三角形

AD ハ頂角 A ヨリ底ノ正中点 D ニ引ケル直線

$AE \perp BC$ トス

(証)　$\because AB^2 = AD^2 + DB^2 + 2BD.DE$

又　$AC^2 = AD^2 + DC^2 - 2CD \cdot DE$

然ルニ　$BD = CD$

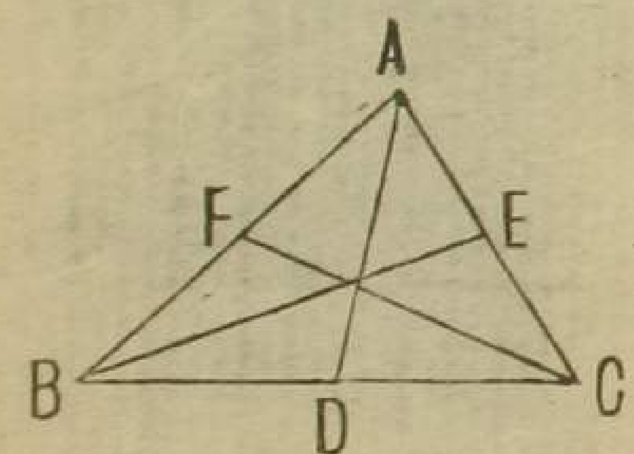
∴　$AB^2 + AC^2 = 2AD^2 + 2DC^2$

---

(4)　ABCハ三角形、

三角点A,B,Cヨリ對邊ノ折半点D,E,FニAD,BE,CFヲ引ク

(証)



∵　$AB^2 + AC^2 = 2BD^2 + 2DA^2$

(3)ニヨル

∴　$2AB^2 + 2AC^2 = BC^2 + 4DA^2$

推シテ　$2AB^2 + 2BC^2 = AC^2 + 4EB^2$

又　$2BC^2 + 2CA^2 = AB^2 + 4FC^2$

上三式ヲ加ヘ相消スレバ

$3AB^2 + 3BC^2 + 3CA^2 = 4DA^2 + 4EB^2 + 4FC^2$

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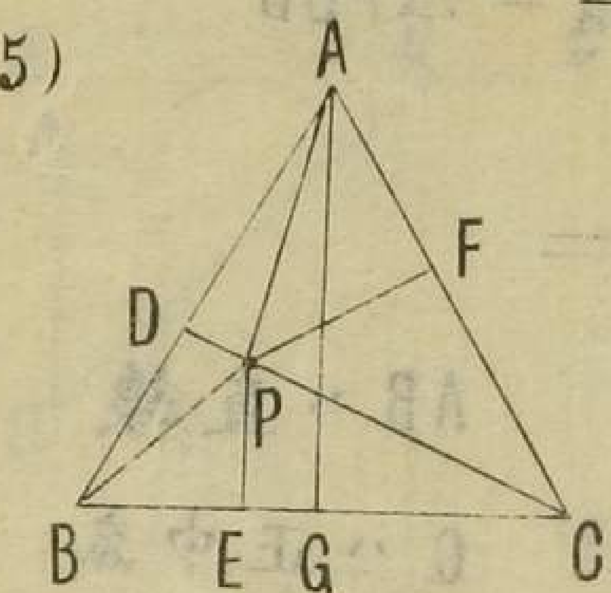
(5)



ABCハ等邊三角形

Pハ形内ノ一点

$PD \perp AB$

$PE \perp BC$,　$PF \perp AC$

$AG \perp BC$

PA, PB, PC. ヲ引ク

(証)　∵ $\triangle ABC = \triangle PAB + \triangle PBC + \triangle PCA$

∴　$AG \cdot BC = AB \cdot PD + AC \cdot PF + BC \cdot PE$

∴　$AG = PD + PF + PE$

P点　E点上ニアル片ハ

$\triangle ABC = \triangle APC + \triangle BPA$

∴　$AG = PD + PF$

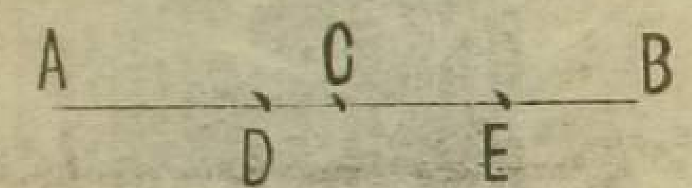
又　P点BCノ外ニ出ル片ハ

$\triangle ABC = \triangle APC + \triangle BPA - \triangle PCB$

$\therefore \quad AC = PD + PF - PE$

---

(6)



AB ハ直線

C ハ正中点

(証)

$\because \quad AD \cdot DB + CD^2 = CB^2$

又 $AE \cdot EB + CE^2 = CB^2$

$\therefore \quad AD \cdot DB + CD^2 = AE \cdot EB + CE^2$

$\therefore \quad CD > CE$ ナレバ

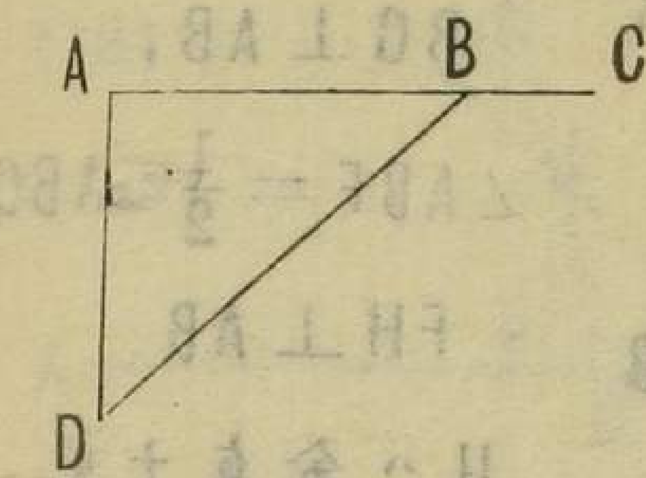
$AD \cdot DB < AE \cdot EB$

又 $CD < CE$ ナレバ

$AD \cdot DB > AE \cdot EB$

---

(7)　AB ＝ 直線



(画法)

AD ≐ AB;

AC ＝ BD ナラシム

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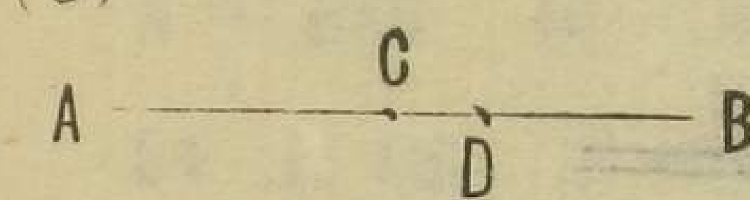
(8)



AB ＝ 直線

C ＝ 折半点

(証)

$\because \quad CD^2 + AD \cdot DB = CB^2$

$\therefore \quad AB^2 = 4CD^2 + 4AD \cdot DB$

---

(9)　直線ヲ等分スヘシ

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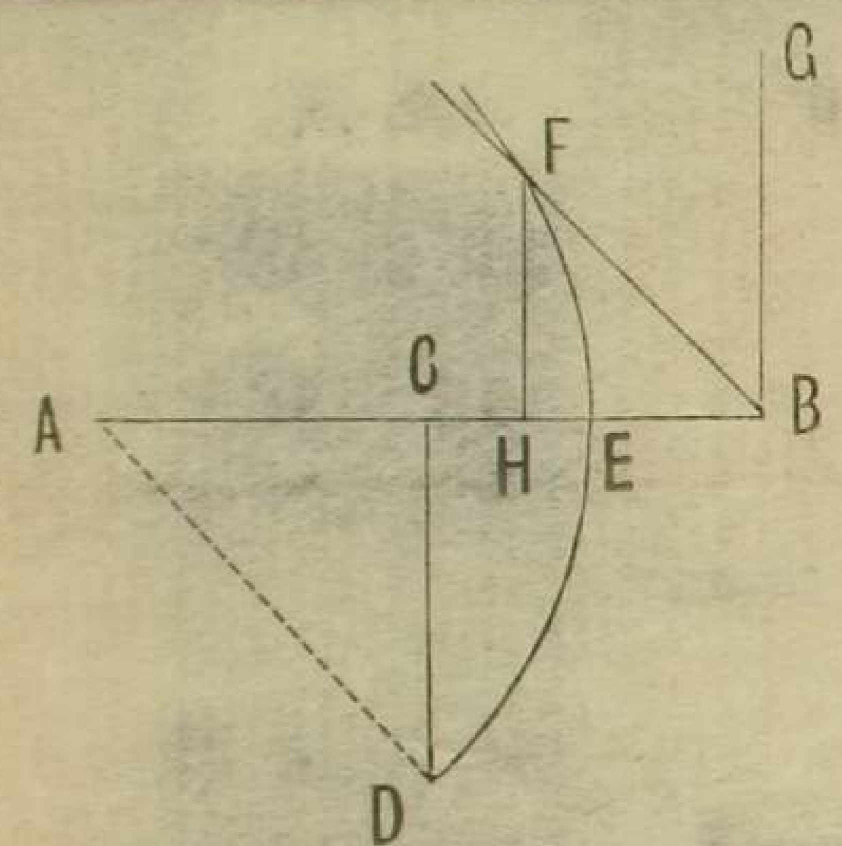
(10)　AB ＝ 甲線　AC ＝ 乙線

(画法)　CD ≐ AC　A ＝ DEF 圏ノ中心



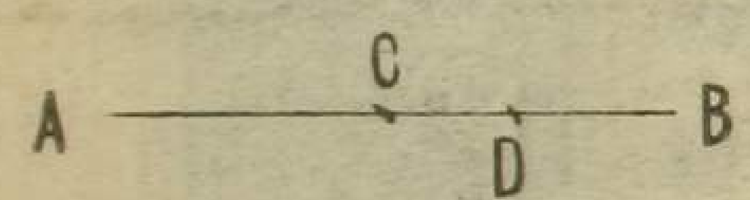
$BG \perp AB$,

$\angle ABF = \frac{1}{2} \angle ABG$

$FH \perp AB$

Hハ分点ナリ

(11)



AB = 直線

C = 甲點

D = 乙點

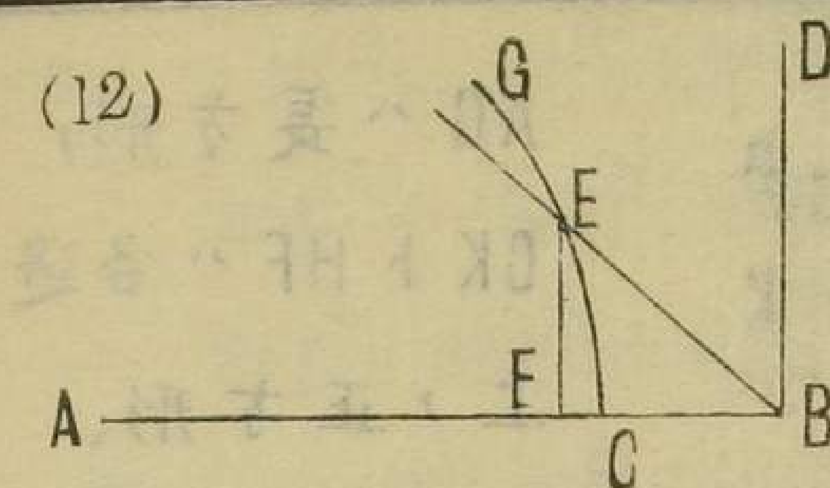
(証)

$\because \quad AD^2 - DB^2 = (AD + DB)(AD - DB)$

然ルニ $AD + DB = AB$; $AD - DB = 2CD$

$\therefore \quad AD^2 - DB^2 = 2AB \cdot CD$

(12)



AB = 直線

AC = 方形ノ一邊

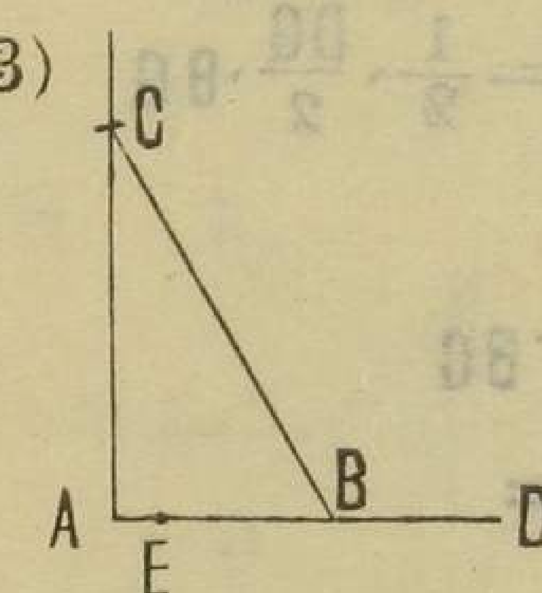
(画法)

$BD \perp AB$ $\quad \angle ABE = \frac{1}{2} \angle ABD$

A = CEG 圏ノ中心

$EF \perp AB$; Fハ分點ナリ

(13)



AB = 直線

(画法)

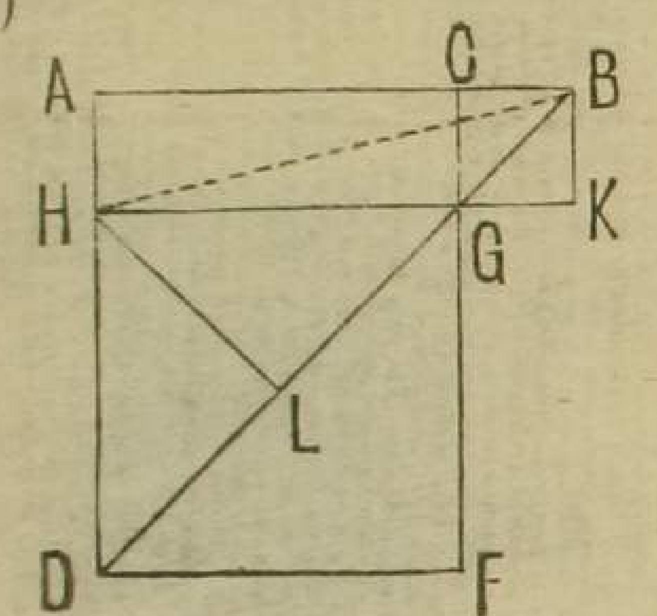
$AC \perp AB$

$BC = 2AB$

$AD = AC \quad BE = BD$

Eハ分點ニシテ $AB^2 + AE^2 = 2BE^2$ ナリ

(14)



AG ハ長方形；CK ト HF ハ各邊上ノ正方形、BG ト GD ハ其對角線

HL ⊥ GD トス

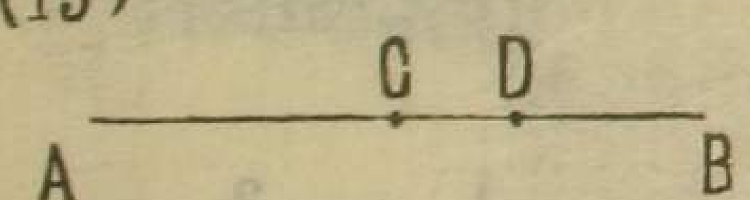
(証) $\triangle BGH = \frac{1}{2} \square AG$

$\triangle BGH = \frac{1}{2} HL \cdot BG = \frac{1}{2}、\frac{DG}{2}、BG$

$\frac{1}{2} \square AG = \frac{1}{2} AC \cdot BC$

$\therefore \frac{1}{2} DG \cdot BG = AC \cdot BC$

(15)



AB ＝ 直線

C ＝ 甲點

D ＝ 乙點

(証) $\because AC^2 = CD^2 + AD \cdot BD$

$2AC^2 = 2CD^2 + 2AD \cdot BD$

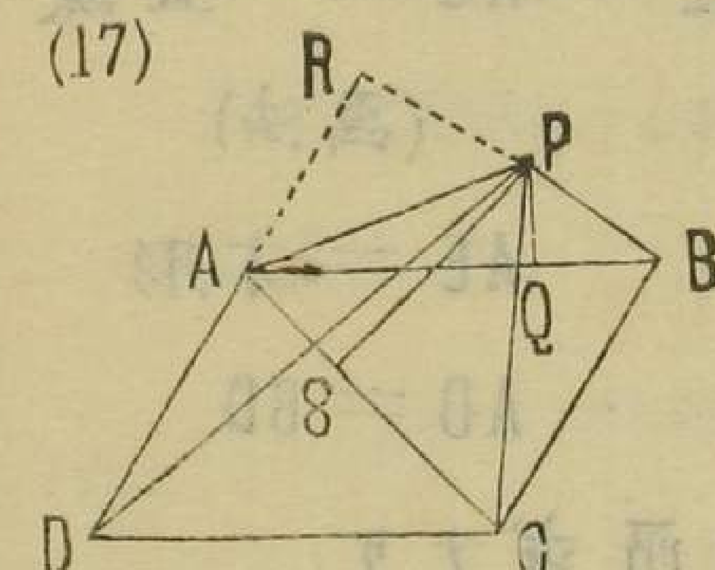
$\therefore 2AC^2 + 2CD^2 = 4CD^2 + 2AD \cdot BD$

然ルニ $AD^2 + BD^2 = 2AC^2 + 2CD^2$

$\therefore AD^2 + BD^2 = 4CD^2 + 2AD \cdot BD$

(16) ユークリッド第二巻ノ七ヲ見ルベシ

(17)



此圖ハ第一巻問題七ト相同シ故ニ

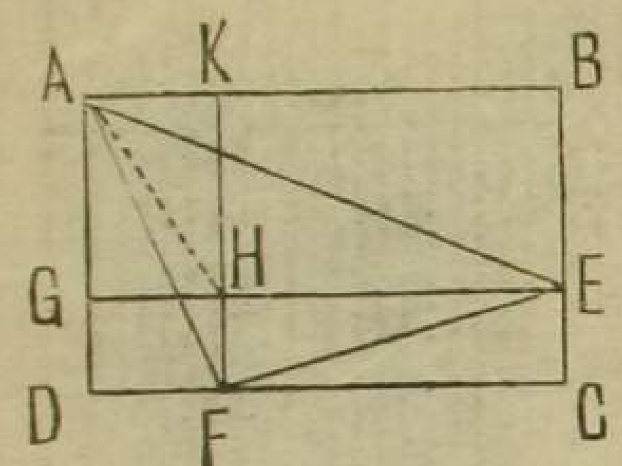
(証)

$\triangle PAC = \triangle PAD \pm \triangle PAB$

$\therefore \frac{1}{2} P8 \cdot AC = \frac{1}{2} PQ \cdot AB \pm \frac{1}{2} BR \cdot AD$

$\therefore P8 \cdot AC = PQ \cdot AB \pm PR \cdot AD$

(18)



(証)

$\because\ 2\triangle AHE = \square BEHK$

$2\triangle AHF = \square DFHG$

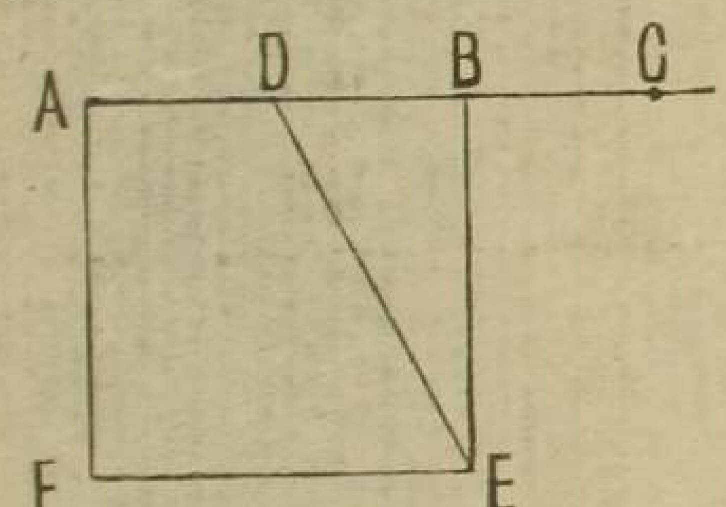
$2\triangle EFH = \square CEHF$

上ノ三式ヲ相加フレバ

$2\triangle AEF = \ulcorner BCDGHK$

$\therefore\ 2\triangle AEF + DF\cdot BE = \square AC$

(19)



AB＝一直線

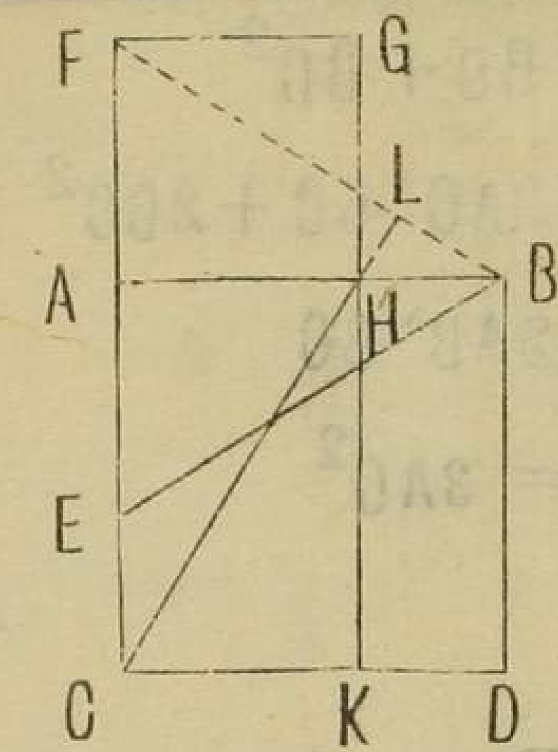
(画法)

AE＝方形

AD＝BD

DC＝DE；　BC＝所求ナリ

(20)　AFGH＝方形　ABDC＝方形



AE＝CE

(証)

$\because\ AC = AB;\ AH = AF$

$\therefore\ AC, AH$ ハ各＝AB, AF

$\angle HAC = \angle BAF$

$\therefore\ CH = BF;\ \angle CHA = \angle AFB$

$\angle HCA = \angle ABF;\ \angle BHL = \angle AHC = \angle AFB$

$\angle ABE + \angle AFB = r.a.$

$\therefore\ \angle BHL + \angle HBL = r.a.$

$\therefore\ \angle HLF = r.a.\quad \therefore\ CL \perp BF$

(21)

(証)

$\because\ AC^2 = AB\cdot BC$

$\therefore\ AC^2 + BC^2 + 2AC\cdot BC = AB\cdot BC + 2AC\cdot BC + BC^2$

$\therefore\ AB^2 = AB\cdot BC + 2AC\cdot BC + BC^2$

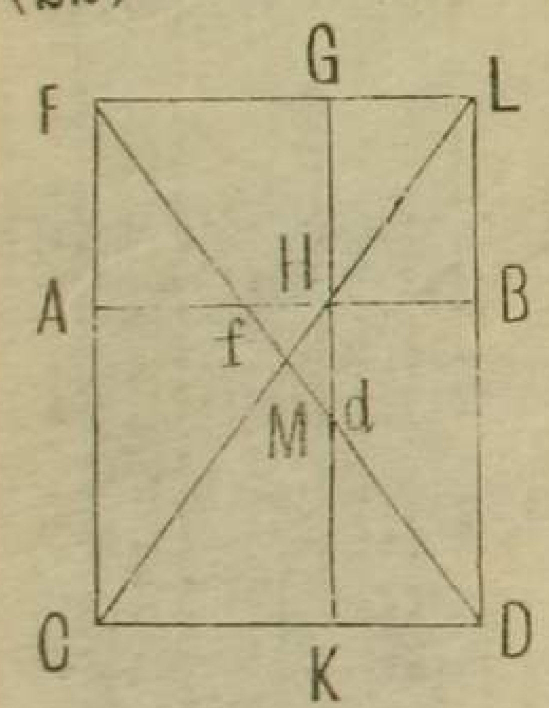
$\therefore\ AB^2 + BC^2 = AB\cdot BC + 2AC\cdot BC + 2BC^2$

$= AB\cdot BC + 2AB\cdot BC$

$= 3AB\cdot BC = 3AC^2$

$\therefore\ AB^2 + BC^2 = 3AC^2$

(22)



FG, DBヲ延バシLニ會セシメLCヲ引ケハHヲ貫クベシ且ッMニ於テFDヲ折半ス

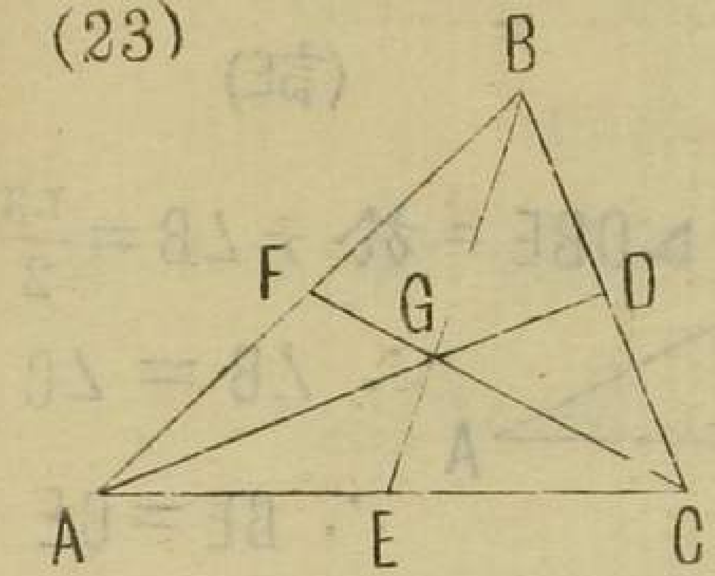
$\because\ \triangle FAf \cong \triangle LBH$

$\therefore\ Ff = LH \quad \therefore\ fM = MH$

然ルニ $fHd = r.a.$ $\therefore\ MH = Md = Mf$

然ルニ $FM = MD$ $\therefore\ Ff = dD$

(23)



(証)

$\because\ AG = 2GD$

$BG = 2GE$

$CG = 2GF$

$AG^2 + BG^2 = 2FG^2 + 2AF^2 \ (\triangle ABG)$

$BG^2 + CG^2 = 2DG^2 + 2BD^2 \ (\triangle BCG)$

$CG^2 + AG^2 = 2EG^2 + 2AE^2 \ (\triangle ACG)$

上三式相加ヘテ二倍スレハ

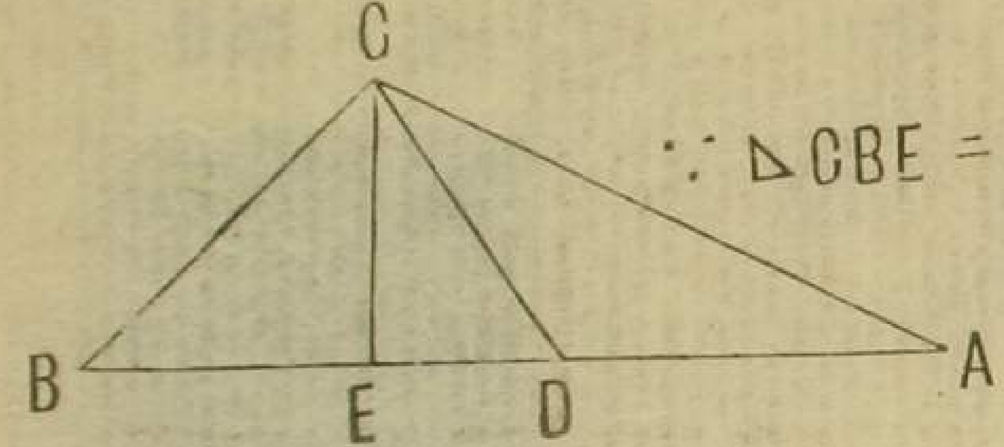
$4AG^2 + 4BG^2 + 4CG^2 = 4FG^2 + 4DG^2 + 4EG^2 + 4AF^2 + 4BD^2 + 4AE^2$

$\therefore\ 4AG^2 + 4BG^2 + 4CG^2 = AG^2 + BG^2 + CG^2 + AB^2 + BC^2 + AC^2$

$\therefore\ 3AG^2 + 3BG^2 + 3CG^2 = AB^2 + BC^2 + CA^2$

$\therefore\ AG^2 + BG^2 + CG^2 = \frac{AB^2 + BC^2 + CA^2}{3}$

(24)



(証)

$\because \triangle CBE$ ニ於テ $\angle B = \frac{r.a.}{2}$

$\therefore \angle B = \angle C$

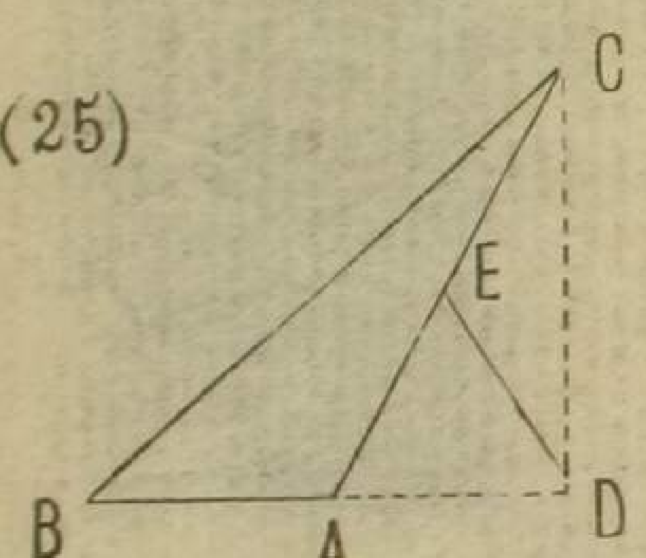
$\therefore BE = CE$

$\therefore AC^2 + BC^2 = 2AD^2 + 2CD^2$

$\therefore AC^2 + 2CE^2 = 2AD^2 + 2DE^2 + 2CE^2$

$\therefore AC^2 = 2AD^2 + 2DE^2$

(25)



$\because \angle BAC = \frac{2}{3}(2\ r.a.)$

$CD \perp BD$

AC ヲ E点ニ於テ折半シ

DE ヲ引ケバ

$DE = AE = EC = \frac{AC}{2}$

$\therefore \angle EAD = \angle EDA = \frac{1}{3}(2r.a.)$

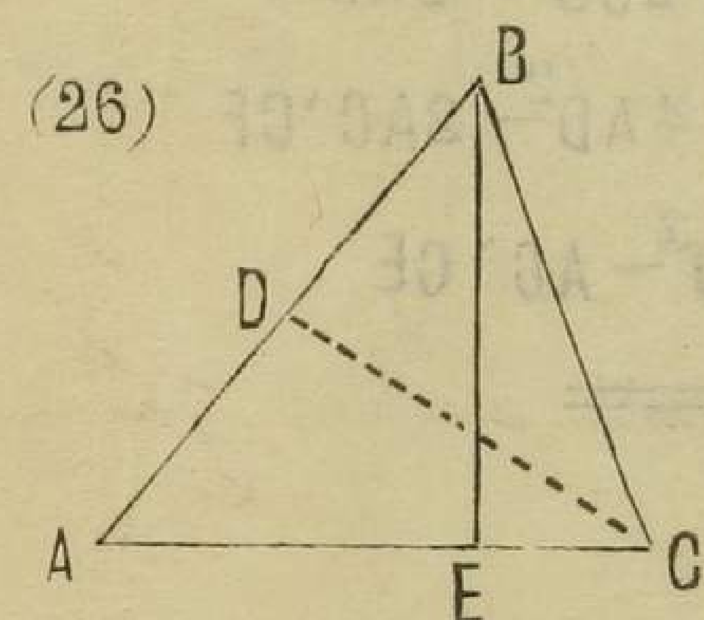
$\therefore \angle AED = \frac{1}{3}(2r.a.)$ $\therefore AE = AD$

$\therefore 2AD = AC$

然ルニ $BC^2 = AB^2 + AC^2 + 2AB \cdot AD$

$\therefore BC^2 = AB^2 + AC^2 + AB \cdot AC$

(26)



ABC ハ三角形

BE ハ垂線

$BD = DA$

(証)

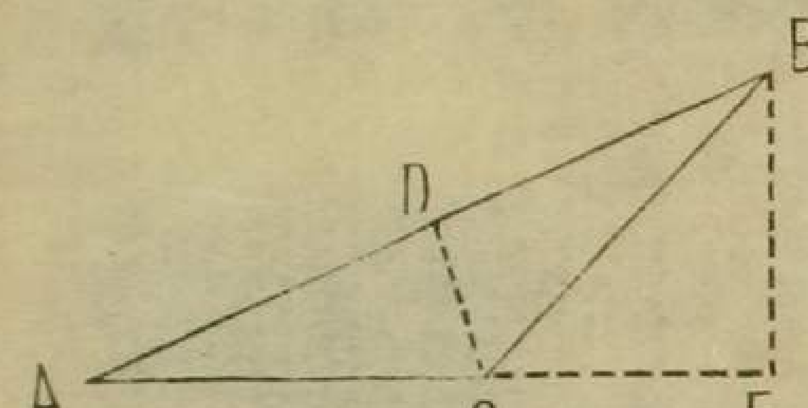
$\because AC^2 + CB^2 = AB^2 + 2AC \cdot CE$

$AC^2 + CB^2 = 2CD^2 + 2AD^2$

$\therefore 2CD^2 + 2AD^2 = 4AD^2 + 2AC \cdot CE$

$\therefore 2CD^2 = 2AD^2 + 2AC \cdot CE$

$\therefore CD^2 = AD^2 + AC \cdot CE$



上圖ニ同シ

（証）

$\because AC^2 + CB^2 = AB^2 - 2AC \cdot CE$

$AC^2 + CB^2 = 2CD^2 + 2AD^2$

$\therefore 2CD^2 + 2AD^2 = 4AD^2 - 2AC \cdot CE$

$\therefore CD^2 = AD^2 - AC \cdot CE$

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(27)　(23)ニ同シ

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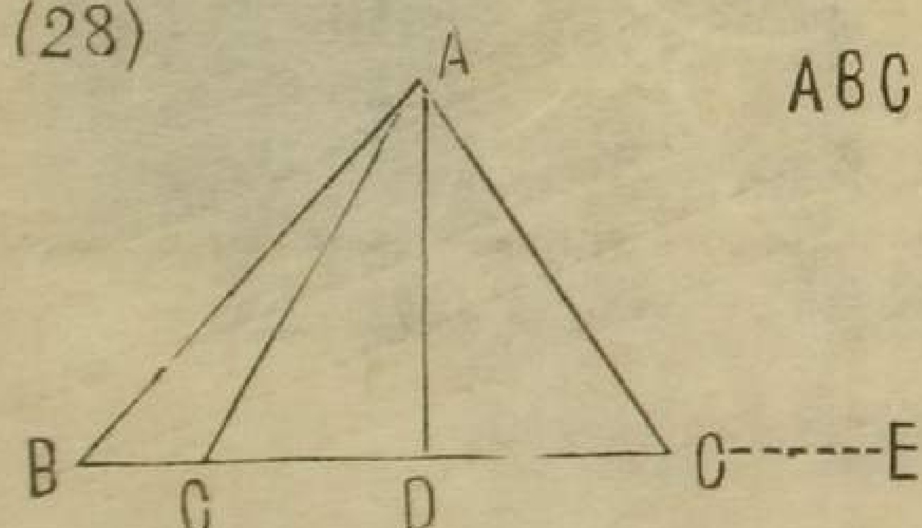
(28)



ABCハ不等辺三角形、

$AD \perp BC$

$AB > AC$

（画法）

BCヲ延シ $CE = BD = CD$ ナラシムベシ

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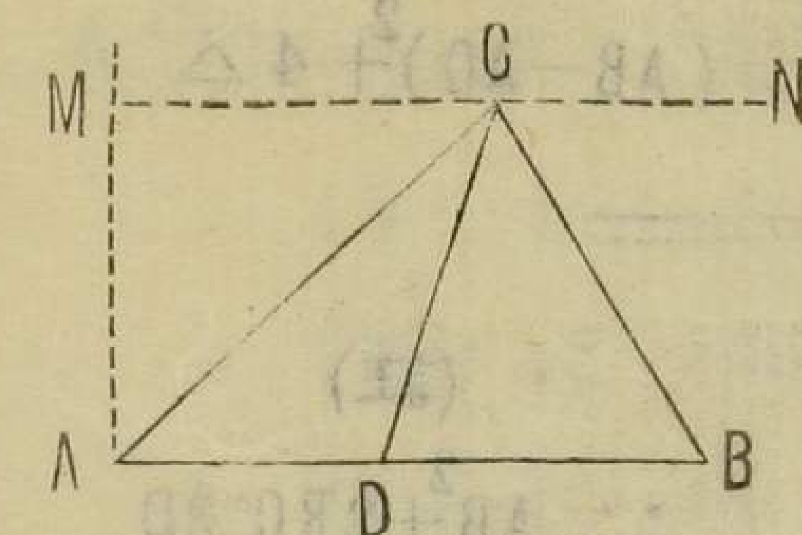
(29)　（注意）

本文積ト底ヲ前知スルトアルヲ以テ底ト高トヲ前知シタルモノト定ムベシ



（画法）

$AB =$ 底

$AM =$ 高

$AM \perp AB$

$MN \parallel AB$

ABヲDニ於テ折半シ底ヲ折半スル線ノ長ヲ半徑トシDヲ中心トシテ圏ヲ画キC點ニ於テMNヲ切ラシメAC，CBヲ引ケバ△ABCハ所要ノ三角形ナリ

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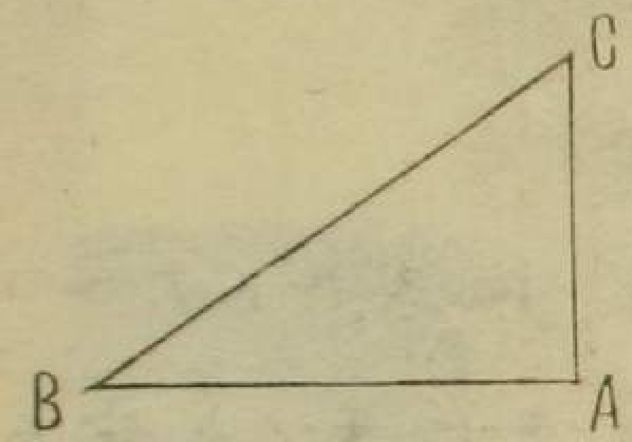
(30)　（証）$\because BC^2 = AB^2 + AC^2$



然ルニ $(AB-AC)^2$

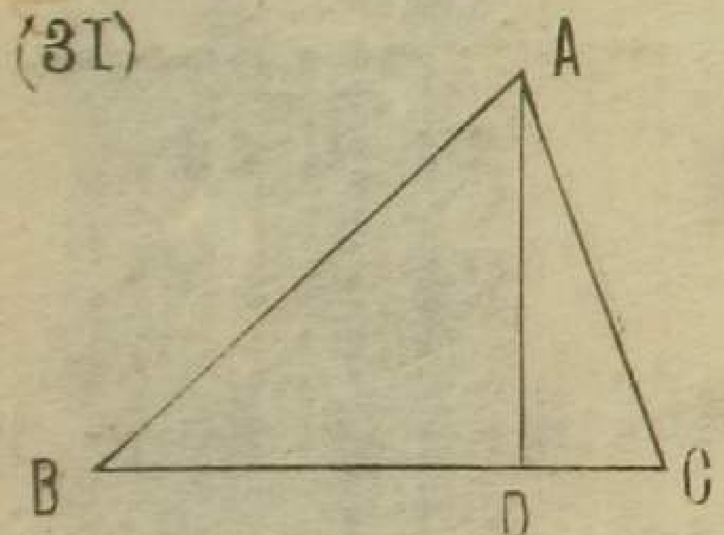
$= AB^2 + AC^2 - 2AB \cdot AC$

$4\triangle = 2AB \cdot AC$

$\therefore\ (AB-AC)^2 + 4\triangle = AB^2 + AC^2$

$\therefore\ BC^2 = (AB-AC)^2 + 4\triangle$

(31)



(証)

$\because\ AB^2 + 2BC \cdot CD$

$= AC^2 + BC^2$

$\therefore\ 2AB^2 + 2BC \cdot CD$

$= AB^2 + BC^2 + AC^2$

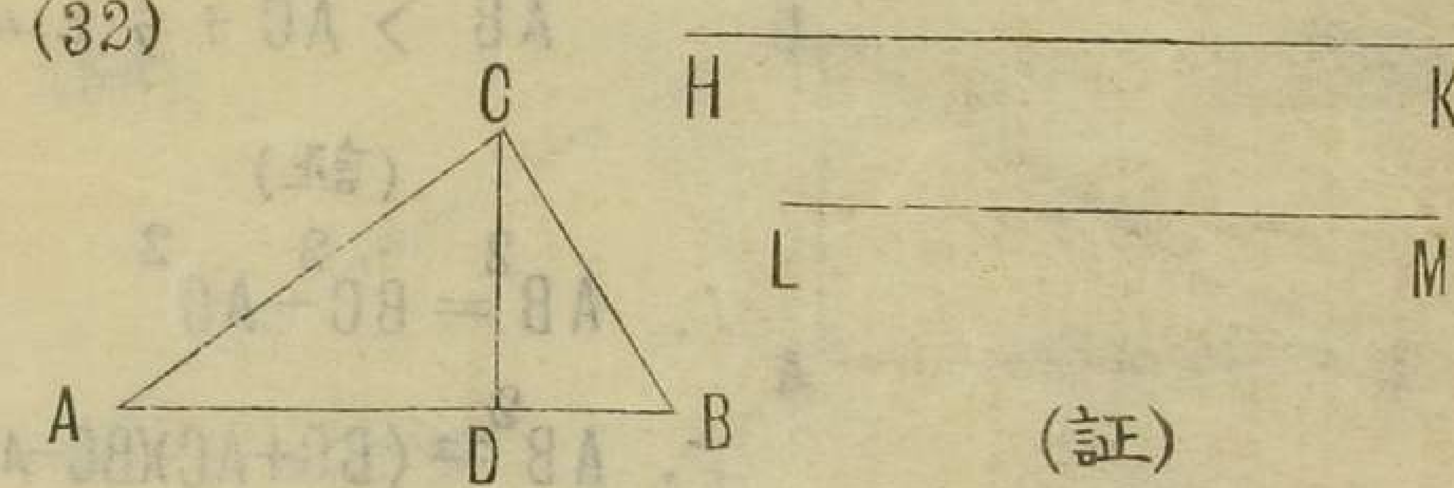
又 $AC^2 + 2BC \cdot BD = AB^2 + BC^2$

$\therefore\ 2AC^2 + 2BC \cdot BD = AB^2 + BC^2 + AC^2$

$\therefore\ 2AC^2 + 2BC \cdot BD = 2AB^2 + 2BC \cdot CD$

$\therefore\ AC^2 + BC \cdot BD = AB^2 + BC \cdot CD$

(32)



(証)

$\because\ LM = AB + CD$

$\therefore\ LM^2 = AB^2 + 2AB \cdot CD + CD^2$

又 $HK = AC + CB$

$\therefore\ HK^2 = AC^2 + CB^2 + 2AC \cdot CB$

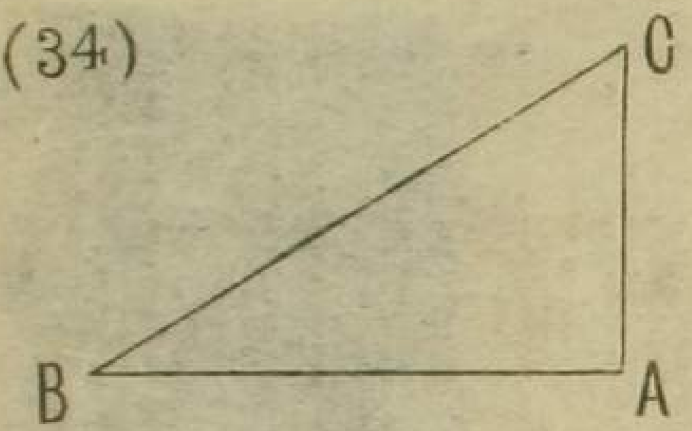
然ルニ $\left.\begin{array}{l} AC^2 + CB^2 = AB^2 \\ AC \cdot CB = CD \cdot AB \end{array}\right\}$ ナリ

$\therefore\ HK^2 + CD^2 = AB^2 + 2AB \cdot CD + CD^2$

$\therefore\ HK^2 + CD^2 = LM^2$

(33)　ABヲ折半シ此ニP點ヲ設クベシ

(34)



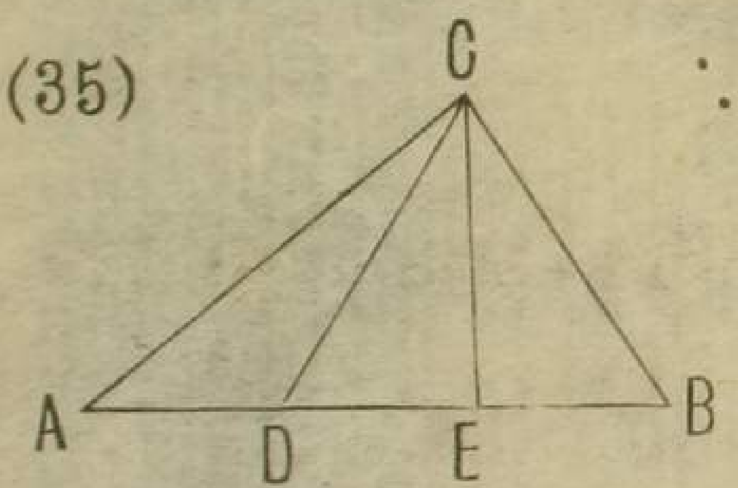
AB > ACナラシム

(証)

$\therefore\ AB^2 = BC^2 - AC^2$

$\therefore\ AB^2 = (BC+AC)(BC-AC)$

(35)



(証)

$\because\ AC^2 + CE^2 = 2CD^2 + 2DE^2$

$CB^2 + CD^2 = 2CE^2 + 2DE^2$

上二式ヲ加フレバ

$AC^2 + CB^2 = CD^2 + CE^2 + DE^2 + 3DE^2$

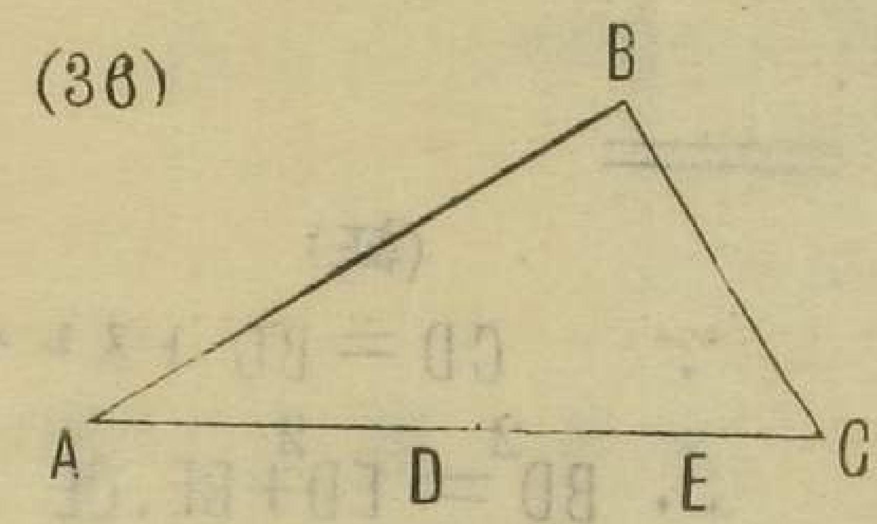
然ルニ $AC^2 + CB^2 = AB^2$

$AB^2 = 9DE^2 \quad \therefore\ 3DE^2 = \frac{1}{3}AB^2$

$\therefore\ AB^2 = CD^2 + CE^2 + DE^2 + \frac{1}{3}AB^2$

$\therefore\ \frac{2}{3}AB^2 = CD^2 + CE^2 + DE^2$

(36)



ABCハ直三角形

$\angle B = $ r.a.

$CD = CB$

$AE = AB$

(証)

$\therefore\ AC^2 = AE^2 + CE^2 + 2AE \cdot EC$

然ルニ $AE \cdot EC = AD \cdot EC + DE \cdot EC$

$AC^2 = AB^2 + BC^2 = AE^2 + CD^2$

$\therefore\ AE^2 + CD^2 = AE^2 + CE^2 + 2AD \cdot EC + 2DE \cdot EC$

$\therefore\ CD^2 = CE^2 + 2DE \cdot EC + 2AD \cdot EC$

上式ノ左右ニDE^2ヲ加フレバ

$CD^2 + DE^2 = CE^2 + 2DE \cdot EC + DE^2 + 2AD \cdot EC$

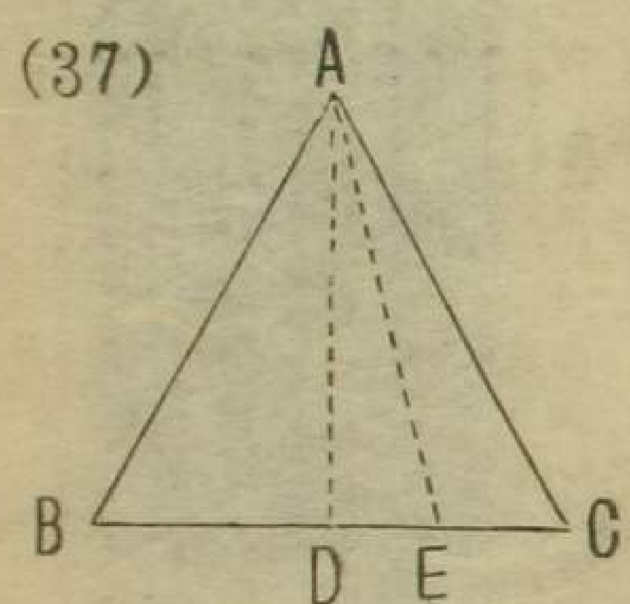
$\therefore\ CD^2 + DE^2 = CD^2 + 2AD \cdot EC$

$\therefore\ DE^2 = 2AD\cdot EC$

即中片ノ正方形ハ外片ノ長方形ノ二倍ニ等シ

(証)

(37)



$\because\ CD = BD$ トスレハ

$\therefore\ BD^2 = ED^2 + BE.CE$

$\therefore\ BD^2 + AD^2 = ED^2 + AD^2 + BE\cdot CE$

$\therefore\ AB^2 = AE^2 + BE.CE$

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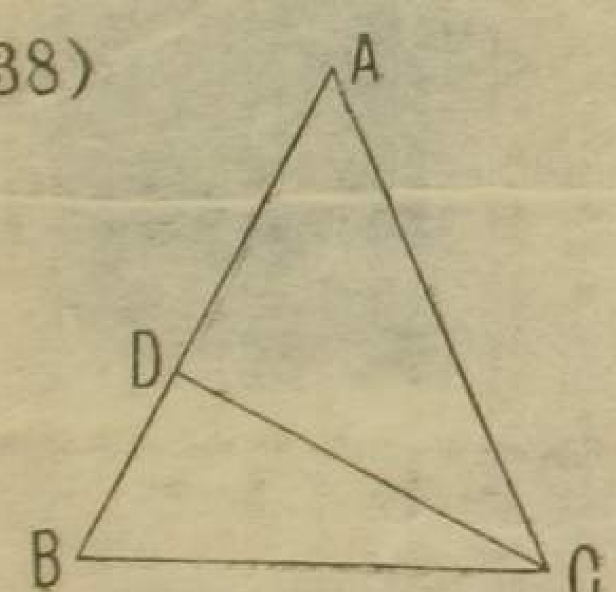
(38)



ABC ハ二等邊三角形

BC ハ底

$CD \perp AB$

(証) $\because\ AC^2 + 2AB\cdot BD = AB^2 + BC^2$

然ルニ $AC^2 = AB^2$ ナリ

$\therefore\ 2AB\cdot BD = BC^2$

$\therefore\ AB\cdot BD = \frac{BC^2}{2}$

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(39) 上題ニ於テ (証)

$BC^2 = 2AB\cdot BD$ ナルヲ証セリ

然ルニ $AB\cdot BD = AD\cdot BD + BD^2$

$\therefore\ BC^2 = 2AD\cdot BD + 2BD^2$

然ルニ $BC^2 = CD^2 + BD^2$

$\therefore\ CD^2 + BD^2 = 2AD\cdot BD + 2BD^2$

$\therefore\ CD^2 = BD^2 + 2AD\cdot BD$

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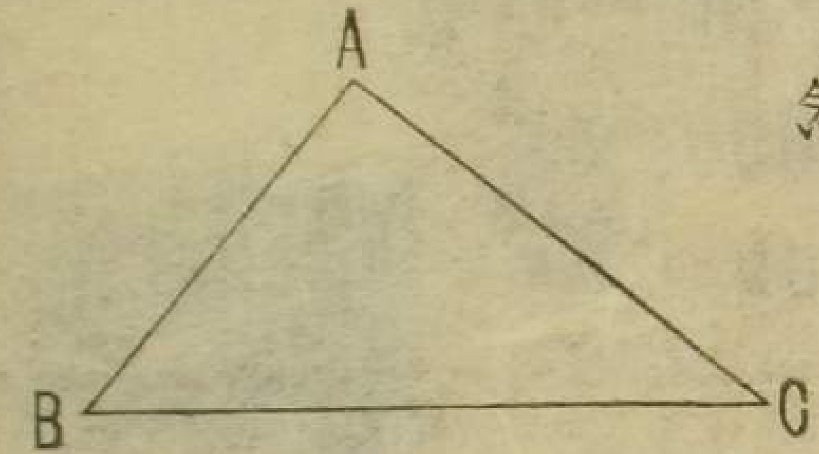
(40) $\angle A =$ r.a. BC = 底

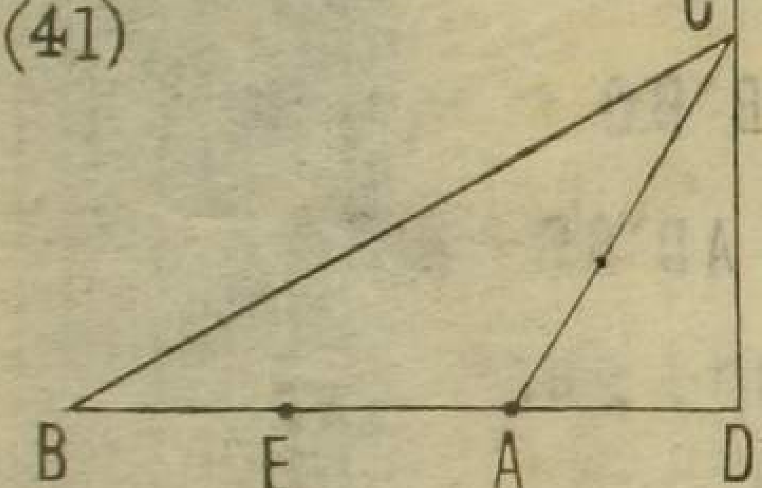
(証) $\because\ BC^2 = BA^2 + AC^2 = 2AB^2$



然ルニ積 $= \frac{AB^2}{2}$

$\therefore BC^2 = 4$ 積

(41)



（画法）

直線DBヲ引キ

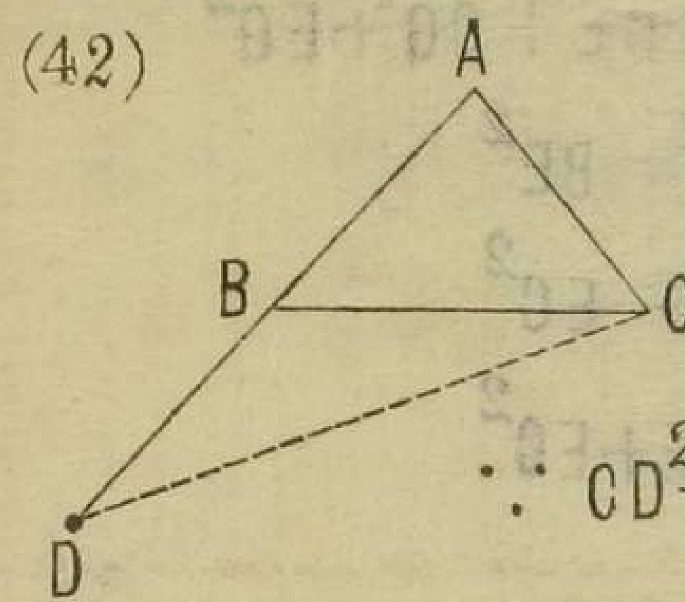
DC⊥DBニ引キ

BDヲ三等分シ

D點ニ近キ分點Aヲ中心トシABニ等シキ半徑ヲ以テ圏ヲ画キDCヲC點ニ於テ切リ

CA, CBヲ引ケハ△ABCヲナス即チ所要ノ三角形ナリ

(42)



$AB = AC$

$AB = BD$

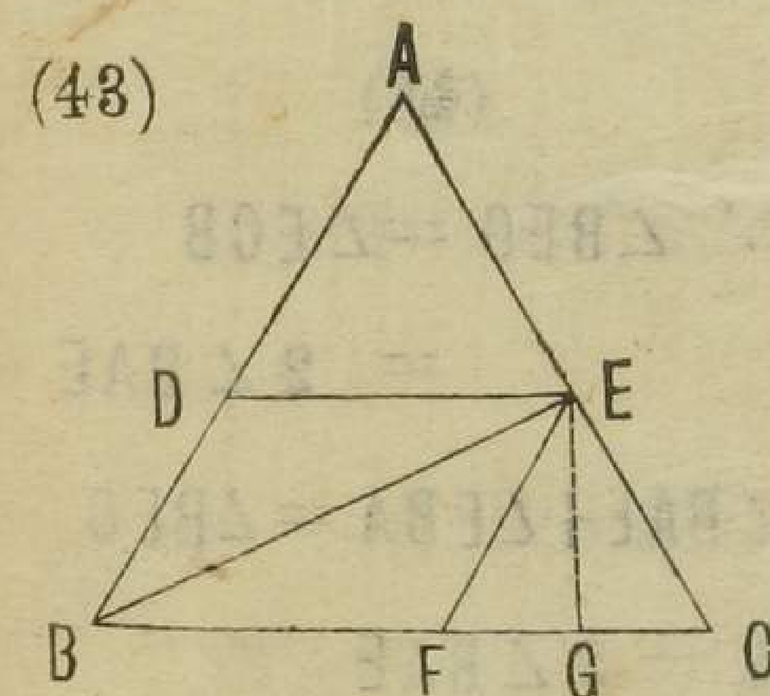
（証）

$\because CD^2 + AC^2 = 2BC^2 + 2AB^2$

然ルニ $AC = AB$

$\therefore CD^2 = AB^2 + 2BC^2$

(43)



EF ∥ DB

EG ⊥ BC

（証）

$\because DE = BF$

$DB = EF = EC$

$FG = GC$

$\because BG^2 = CB.BF + FG^2$

$\therefore BG^2 = CB.DE + CG^2$

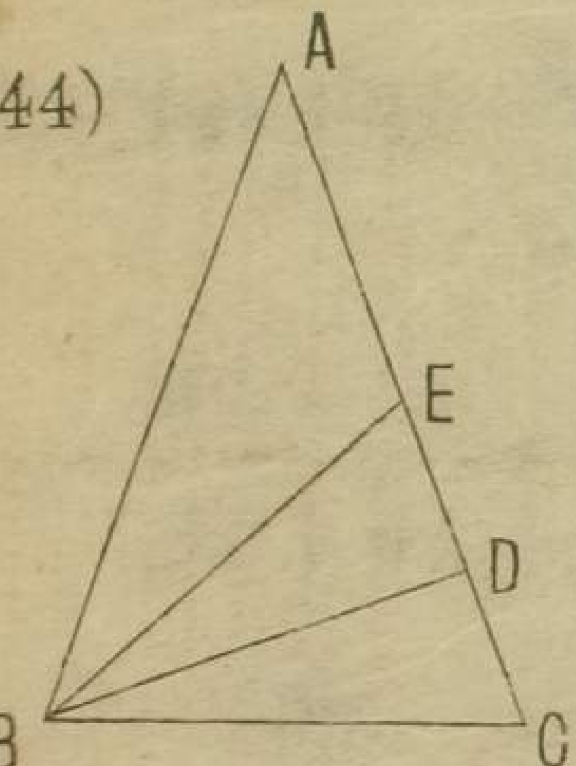
∴ $BG^2+EG^2=CB.DE+CG^2+EG^2$

然ルニ　$BG^2+EG^2=BE^2$

$CG^2+EG^2=EC^2$

∴ $BE^2=CB.DE+EC^2$

(44)



$BD\perp AC$

$BE=BC$

(証)

∵ $\angle BEC=\angle ECB$

$=2\angle BAE$

$\angle BAE+\angle EBA=\angle BEC$

∴ $\angle BAE+\angle EBA=2\angle BAE$

∴ $\angle EBA=\angle BAE$

∴ $AE=EB=BC$

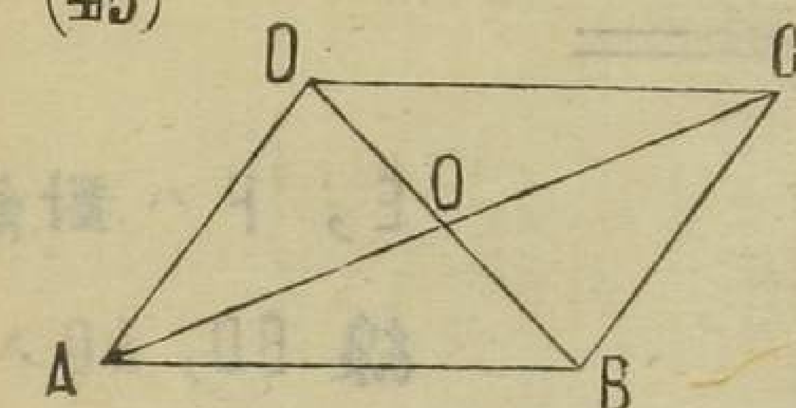
又 ∵ $AB^2=BE^2+AE^2+2AE\cdot ED$

∴ $AC^2=BC^2+AE^2+AE\cdot EC$

然ルニ $AE^2+AE\cdot EC=AE\cdot AC=AC\cdot BC$

∴ $AC^2=BC^2+AC\cdot BC$

(45)



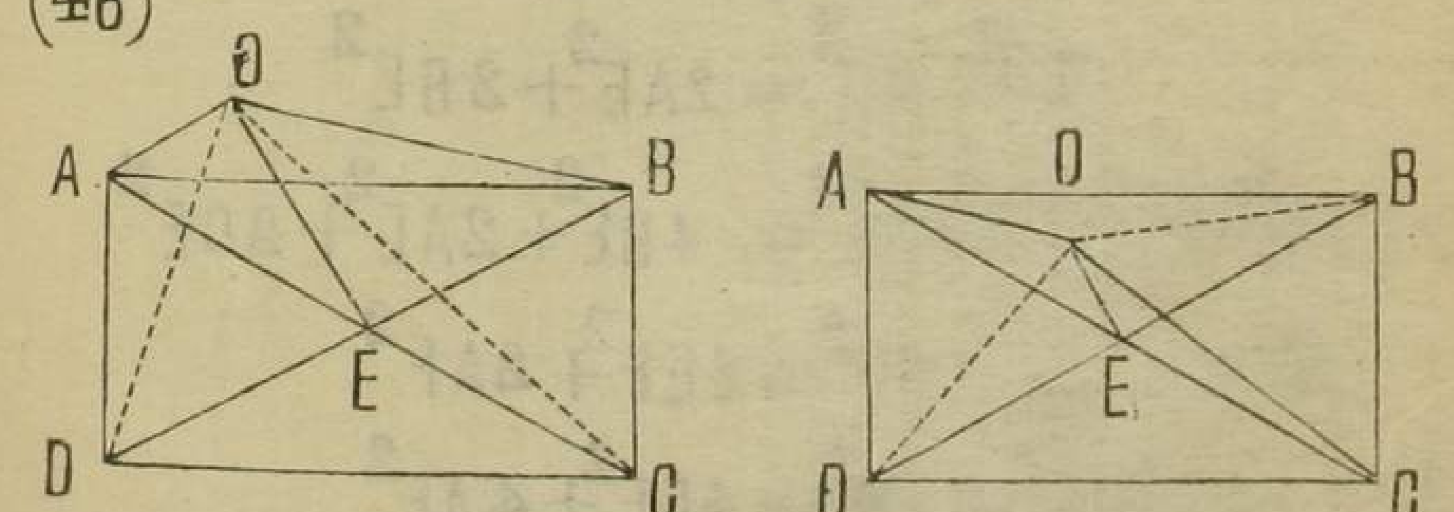
(証)

∵ $DC^2+BC^2=$
$2DO^2+2CO^2.$
$AD^2+AB^2=$
$2DO^2+2AO^2.$

∴ $AB^2+BC^2+CD^2+DA^2=4DO^2+4AO^2$

$=AC^2+BD^2$

(46)



(証) ∵ $AO^2+OC^2=2EO^2+2AE^2$

$BO^2+OD^2=2EO^2+2BE^2$

然ル＝ $AE=BE$

∴ $AO^2+OC^2=BO^2+OD^2$

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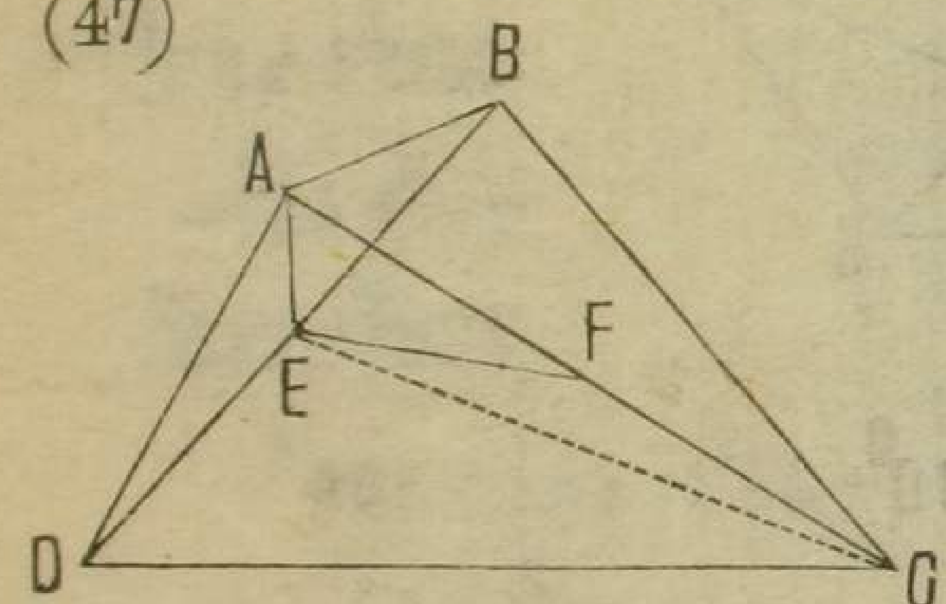
(47)



E，Fハ對角線BD，ACノ正中點

(証)

∵ $BC^2+CD^2=2BE^2+2CE^2$

$AB^2+AD^2=2AE^2+2BE^2$

∴ $AB^2+BC^2+CD^2+DA^2=4BE^2+2AE^2+2CE^2$

然ル＝ $AE^2+CE^2=2EF^2+2AF^2$

$2AE^2+2CE^2=4EF^2+4AF^2$

∴ $4BE^2+2AE^2+2CE^2=4BE^2+4AF^2+4EF^2$

$=BD^2+AC^2+4EF^2$

∴ $BD^2+AC^2+4EF^2=AB^2+BC^2+CD^2+DA^2$

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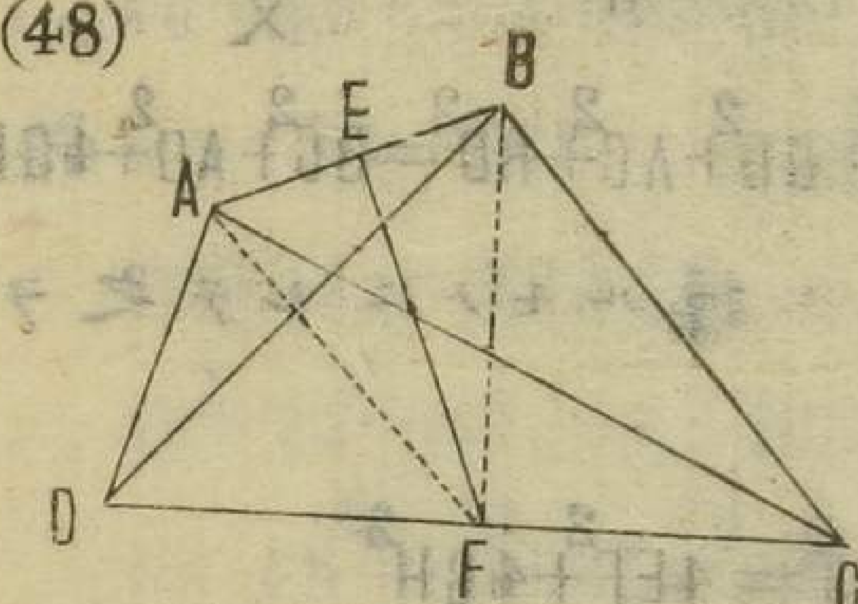
(48)



ABCDハ不等辺四邊形、E，Fハ相對辺AB，CDノ中點

(証) ∵ $AC^2+AD^2=2AF^2+2CF^2$

$BC^2+BD^2=2BF^2+2CF^2$

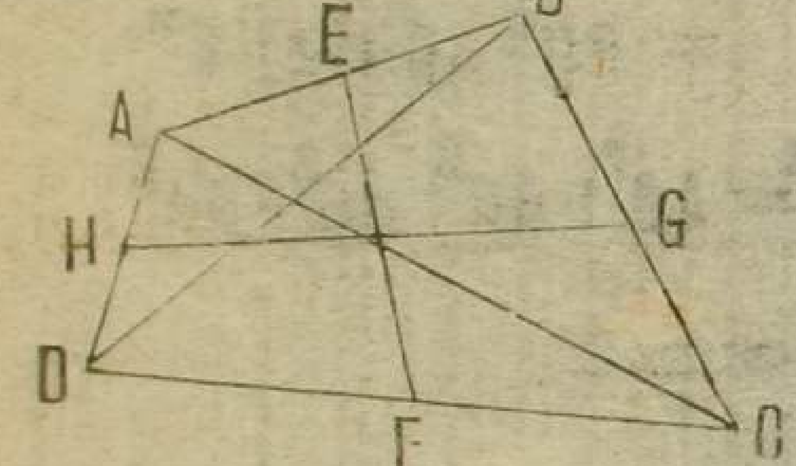
∴ $AD^2+BC^2+AC^2+BD^2=4CF^2+2AF^2+2BF^2$

然ル＝ $AF^2+BF^2=2AE^2+2EF^2$

∴ $AD^2+BC^2+AC^2+BD^2=4CF^2+4AE^2+4EF^2$

$=CD^2+AB^2+4EF^2$

(49)



(証)

$$\therefore AD^2+BC^2+AC^2+BD^2$$
$$=CD^2+AB^2+4FF^2$$

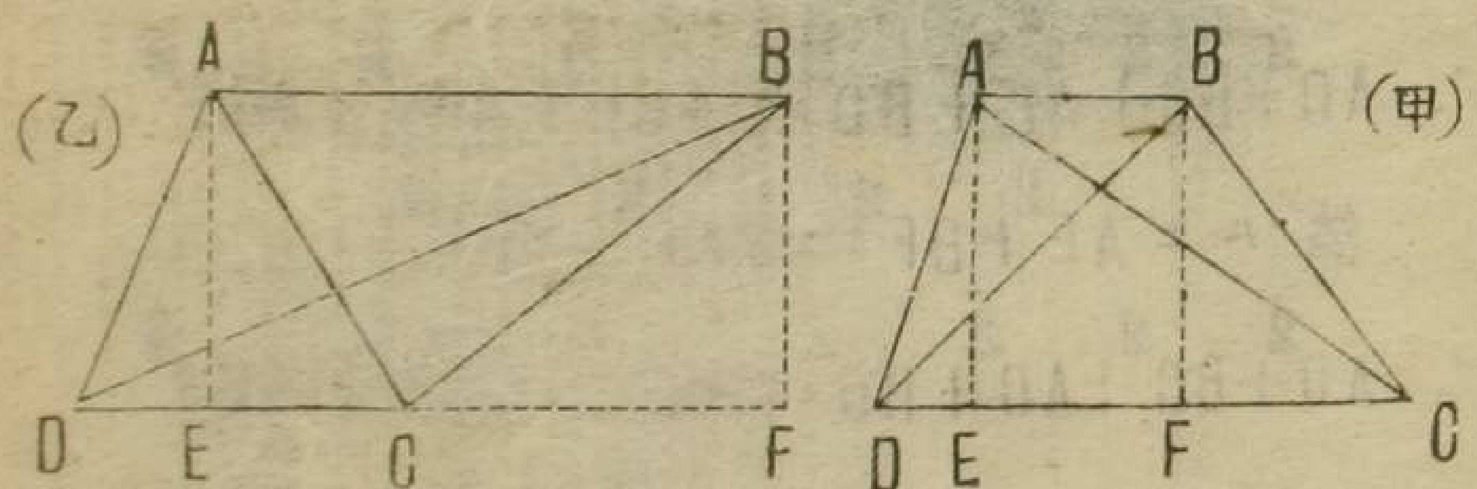
又

$$AB^2+CD^2+AC^2+BD^2=BC^2+AD^2+4GH^2$$

上二式ハ (48) ニ據ルモノニシテ之ヲ相加フレハ

$$2AC^2+2BD^2=4EF^2+4GH^2$$
$$\therefore AC^2+BD^2=2EF^2+2GH^2$$

(50)



$AE \perp CD$, $BF \perp CD$, $AB \parallel CD$.

(証)

$$\therefore AC^2+2CD\cdot DE=AD^2+CD^2$$
$$BD^2+2CD\cdot CF=BC^2+CD^2$$
(甲) 圖ニ據ル

$$\therefore AC^2+BD^2+2CD\cdot DE+2CD\cdot CF=AD^2+BC^2+2CD^2$$

然ルニ $AB=EF$

$$\therefore 2CD\cdot EF=2CD\cdot AB$$
$$\therefore AC^2+BD^2+2CD\cdot DE+2CD\cdot EF+2CD\cdot CF$$
$$=AD^2+BC^2+2CD^2+2CD\cdot AB$$

然ルニ $2CD\cdot DE+2CD\cdot EF+2CD\cdot CF=2CD^2$

$$\therefore AC^2+BD^2+2CD^2=AD^2+BC^2+2CD^2+2CD\cdot AB$$
$$\therefore AC^2+BD^2=AD^2+BC^2+2CD\cdot AB$$

(証)

$$\therefore AC^2+2CD\cdot DE=AD^2+CD^2$$
$$BD^2=BC^2+CD^2+2CD\cdot CF$$
(乙) 圖ニ據ル

$\therefore\ AC^2+BD^2+2CD\cdot DE=AD^2+BC^2+2CD^2+2CD\cdot CF$

$2EF\cdot CD=2AB\cdot CD$ ヲ左右ニ加フレバ

$AC^2+BD^2+2CD\cdot DE+2EF\cdot CD=AD^2+BC^2+2CD^2$
$+2CD\cdot CF+2AB\cdot CD$

然ルニ $2CD\cdot DE+2EF\cdot CD=2CD\cdot DF$

又 $2CD^2+2CD\cdot CF=2CD\cdot DF$

$\therefore\ 2CD\cdot DE+2EF\cdot CD=2CD^2+2CD\cdot CF$

$\therefore\ AC^2+BD^2=AD^2+BC^2+2AB\cdot CD$

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(51) 上圖ヲ用フレハ (証)

$\because\ 2\triangle ACD=AE\cdot CD$

$2\triangle ABC=AE\cdot AB$

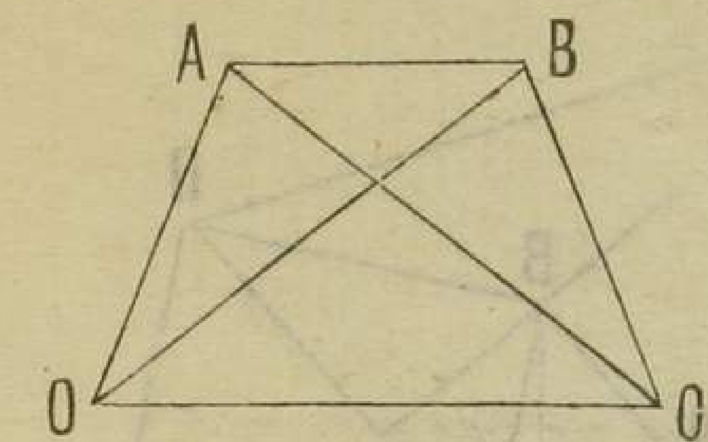
$\therefore\ 2\triangle ACD+2\triangle ABC=AE\,(AB+CD)$

然ルニ ⏢$ABCD=\triangle ACD+\triangle ABC$

$\therefore$ ⏢$ABCD=AE\cdot\frac{AB+CD}{2}$

---

(52)



$AB \parallel DC$

$AD=BC$

(証)

$\because\ AC=BD$

$\therefore\ AC^2=BD^2$

又 (50)ニ於テ

$\because\ AC^2+BD^2=AD^2+BC^2+2AB\cdot CD$

$\therefore\ 2AC^2=2AD^2+2AB\cdot CD$

$\therefore\ AC^2=AD^2+AB\cdot CD$

---

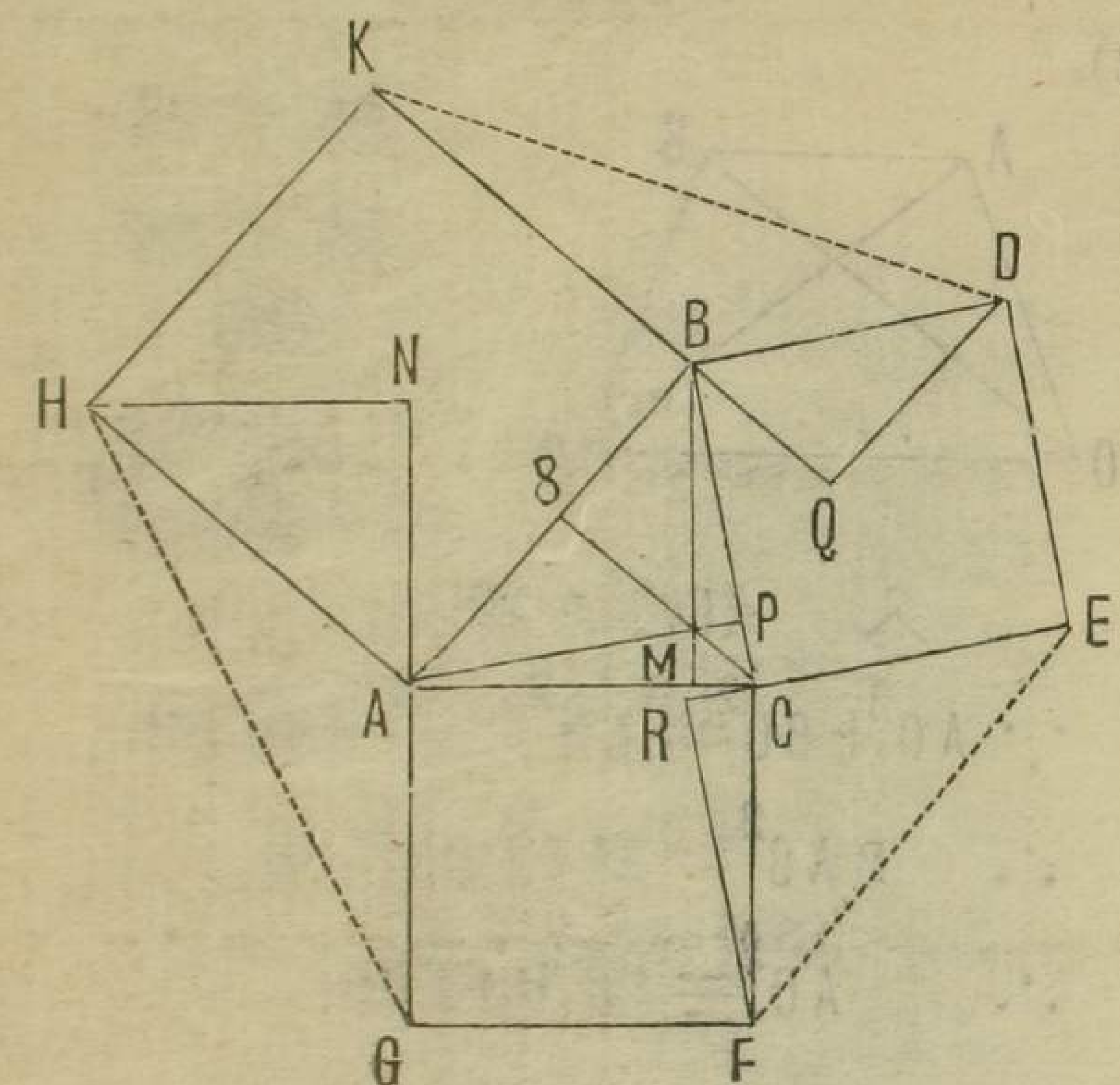
(53) ABC ハ三角形

AK, BE, CG ハ方形

GAN, KBQ, ECR ハ各直線

∠N, ∠Q, ∠R ハ各直角

AP ⊥ BC, BM ⊥ AC, CS ⊥ AB



(証)

$\because \triangle AHN \cong \triangle ABM \quad \therefore AN = AM$

$\triangle BQD \cong \triangle BSC \quad \therefore BQ = BS$

$\triangle CRF \cong \triangle APC \quad \therefore CR = CP$

$\because HG^2 = AG^2 + AH^2 + 2AG \cdot AN$

$\therefore GH^2 = AC^2 + AB^2 + 2AC \cdot AM$

$FE^2 = AC^2 + BC^2 + 2BC \cdot CP$

$DK^2 = AB^2 + BC^2 + 2BA \cdot BS$

$\therefore DK^2 + EF^2 + GH^2 = 2AB^2 + 2AC^2 + 2BC^2 + 2AC \cdot AM + 2BC \cdot CP + 2BA \cdot BS$

然ルニ $BC^2 + 2AC \cdot AM = AB^2 + AC^2$

$AB^2 + 2BC \cdot CP = AC^2 + BC^2$

又 $AC^2 + 2BA \cdot BS = AB^2 + BC^2$

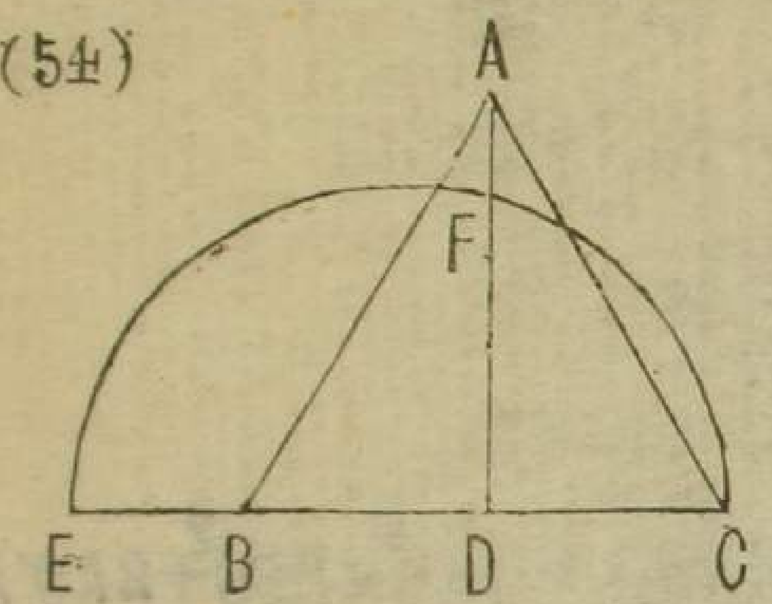
上三式ヲ加フレハ

$2AC \cdot AM + 2BC \cdot CP + 2BA \cdot BS = AB^2 + BC^2 + AC^2$

$\therefore DK^2 + EF^2 + GH^2 = 3AB^2 + 3BC^2 + 3AC^2$

$\therefore DK^2 + ED^2 + EF^2 + FG^2 + GH^2 + HK^2 = 4AB^2 + 4BC^2 + 4AC^2$

(54)



(画法)

等邊三角形ABCノ頂角Aヨリ其底BCノ上ニ垂線ADヲ下シCBヲ引長シDE＝ADニナシCE上ニ半圓EFCヲ画キF點ニ於テADヲ切ラシムレバFD＝△ABCト同積ノ方形ノ一邊ナリ

---

(55) ユークリッド第一巻45ニ據リ本題二形合積ト同積ノ長方形ヲ画キ第二巻14ニ據リテ画法ヲ行フベシ

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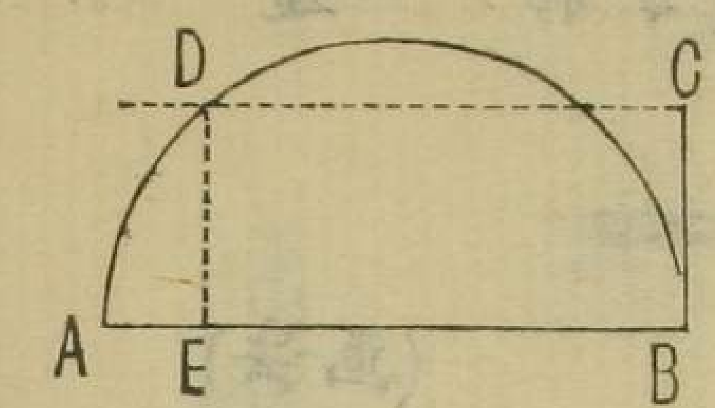
(56) (画法) 分ツベキ直線ABヲ置キ其上ニ半圓ADBヲ画キB點ヨリBCヲ引キ



ABニ直角ナラシメ且ツBCヲシテ前知長方形ト同積ナル方形ノ一邊ト同長ナラシメC點ヨリCDヲ引キABニ平行ナラシメADBヲD點ニ切リDE⊥ABナラシム即チABヲE點ニ於テ分テリ

AE・EB＝前知長方形ナリ

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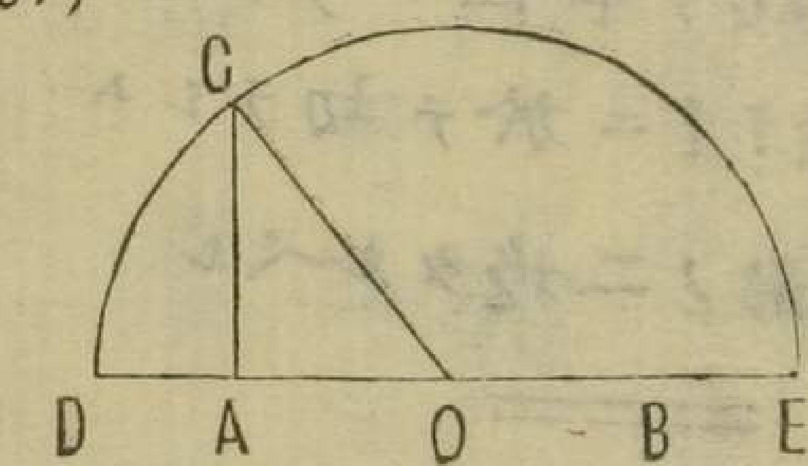
(57)



(画法)

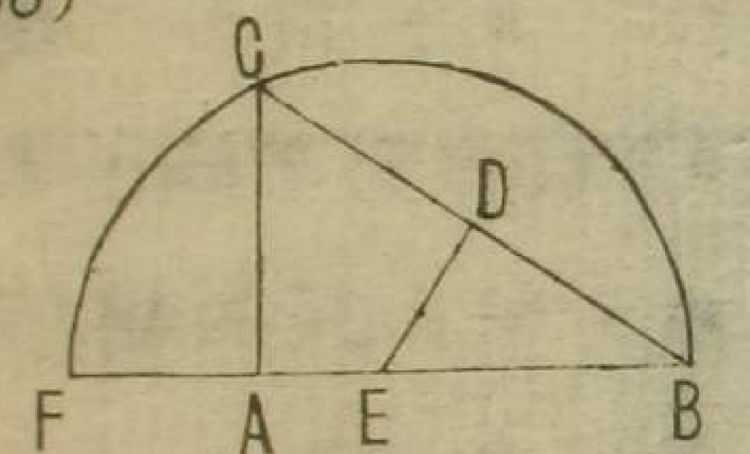
正方形ノ一邊ACト直線ABトAニ於テ直角ヲナサシメ

ABノ正中ニO點ヲ設ケOヲ中心トシOCヲ半徑トシ半圓DCEヲ画ク

AD, AE ハ画クヘキ長方形ノ二邊ニシテ

其差ハ AB ニ等シ

(58)　(画法)



固有一直線ABト固有方形ノ一辺ACトAニ於テ直角ヲナサシメ, CBヲ引キ, Dニ於テCBヲ折半シ, DE⊥CBヲ引キ, ABヲEニ於テ切リ, Eヲ中心トシ, EB又ハECヲ半徑トシ半圓FCBヲ画キ, BAノ延線ヲFニ於テ切ラシム

AB, AF ハ長方形ノ二邊タルベシ

幾何問題解

巻之二終

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