







門 二 2
號
卷



第四篇之一
各式

第一 $u = \frac{ax^3}{3b} + c$

第二 $y = \frac{x^2}{9} + c$

第三 $y = \frac{3x^{\frac{4}{3}}}{4} + c$

第四 $y = 2x^{\frac{1}{2}} + c$

第五 $y = -\frac{1}{2} \frac{1}{x^2} + c$

第六 $y = \frac{a}{3} x^3 + x^{\frac{1}{2}} + c$

第七 $u = \frac{n}{n+1} ax^{\frac{n+1}{n}} + c$

第八 $y = \log(a+x) + c$

第九 $y = 2bc \log(a+bx) + c$

洋算例題續算四篇卷之十一

陸軍大尉福田半編輯

廿三 $u = \frac{e(a+bx^e)^{\frac{1}{2}}}{5a} + c$

廿四 $u = \frac{a}{2}(a^2+x^m)^2 + c$

廿五 $u = (2x+x^e)^{\frac{1}{2}} + c$

式答 = 之 卷

筭 $y = \frac{x}{a} - \frac{x^2}{2a^2} + \frac{x^3}{3a^3} - \frac{x^4}{4a^4} + \frac{x^5}{5a^5} - \dots + c$

筭 $y = \frac{x}{a^2} - \frac{x^2}{4a^4} + \frac{x^3}{7a^7} - \frac{x^4}{16a^8} + \frac{x^5}{25a^5} - \dots + c$

筭 $y = x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots + c$

筭 $\int_a^k 2x dx = k^2 - a^2$

筭 $\int_a^k 3x^2 dx = 152$

筭 $\int_4^6 \frac{\pi}{2} x^2 dx = \frac{76}{3} \pi$

筭 $\int_{10}^{20} 2(l+x) dx = 380$

筭 $y = \frac{a}{2} \log x + c$

士 $y = 3a^2 b \log x + c$

士 $y = a^2 x + abx^e + \frac{1}{3} b^2 x^3 + c$

士 $y = 25x + \frac{76}{3} x^3 + \frac{47}{5} x^5 + c$

士 $y = a^5 x + 3a^2 x^3 + \frac{27}{5} a x^5 + \frac{27}{7} + c$

士 $y = \frac{(a+3x^2)}{3} + c$

士 $y = \frac{e}{3}(x+ax)^{\frac{3}{2}} + c$

士 $y = \frac{1}{2}(2x^3-1)^e + c$

士 $y = \frac{3}{2}(1+x)^{\frac{3}{2}} + c$

士 $y = \frac{1}{3}(ax^e+bx^3)^2 + c$

士 $y = \frac{3}{4}(ax+bx^2)^{\frac{4}{3}} + c$

士 $u = \frac{m(a+bx^e)^{\frac{3}{2}}}{2a} + c$

士 $u = (a^2+x^e)^{\frac{1}{2}} + c$

廿 $y = \frac{1}{\sqrt{6}} \tan^{-1} x \sqrt{\frac{3}{2}} + C$

廿三 $y = \frac{1}{2} \tan^{-1} \frac{3}{2} x + C$

廿四 $y = \frac{1}{\sqrt{5}} \tan^{-1} x \sqrt{5} + C$

廿五 $y = \frac{1}{\sqrt{5}} \tan^{-1} x \sqrt{\frac{2}{3}} + C$

廿六 $y = \sec^{-1} x \sqrt{\frac{3}{2}} + C$

廿七 $y = \frac{5}{\sqrt{2}} \sec^{-1} x + C$

廿八 $y = \frac{1}{3} \sec^{-1} 3x + C$

廿九 $y = \frac{3}{2} \sec^{-1} \frac{2}{9} x + C$

三十 $\int \frac{x^2 dx}{(x+2)^2(x+4)^2} = -\frac{5x+12}{x^2+6x+8} + \log\left(\frac{x+4}{x+2}\right) + C$

卅一 $\int \frac{2x dx}{(x^2+1)(x^2+3)} = \log \sqrt{\frac{x^2+1}{x^2+3}}$

卅二 $u = \frac{1}{3} (x \tan^{-1} \frac{x}{2} - \tan^{-1} x) + C$

卅三 $u = \frac{-1}{2(x-1)^2} - \frac{1}{2} \frac{1}{x-1} + \frac{3}{2} \log \frac{\sqrt{x+1}}{x-1} - \tan^{-1} x + C$

算 $\int_a^b 6(1+xx^2)xx dx = 57600$

算 $\int_a^b \frac{ax}{l+x} = \log \frac{7}{6}$

算 $u = -\log \cos x + C$

卅 $u = \log \sin x + C$

卅一 $u = \tan \frac{1}{2} x + C$

卅二 $u = -\log(1+\cos x) + C$

卅三 $u = \log \tan \frac{1}{2} x + C$

卅四 $u = \tan x - x + C$

卅五 $u = \frac{1}{2} x - \frac{1}{4} \sin 2x + C$

卅六 $y = \sin^{-1} \frac{x}{2} + C$

卅七 $y = 5 \sin^{-1} \frac{x}{\sqrt{3}} + C$

卅八 $y = 2 \cos^{-1} \frac{x}{6} + C$ 卅九 $y = \sin^{-1} x \sqrt{\frac{2}{3}} + C$

卅十 $y = \frac{1}{2} \cos^{-1} \frac{2}{3} x + C$

$$\text{廿八} \quad u = \frac{3}{2b^3} \left\{ \frac{(a+bx^2)^{\frac{11}{2}}}{11} - \frac{2a(a+bx^2)^{\frac{9}{2}}}{9} + \frac{a^2(a+bx^2)^{\frac{7}{2}}}{7} \right\} + C$$

$$\text{廿九} \quad u = -\frac{1}{3}(a+x^2)(a-x^2)^{\frac{5}{2}} + C$$

$$\text{三十} \quad u = \left(\frac{a^2+x^2}{2}\right)^2 - a^2(a^2+x^2) + \frac{a^2}{x} \log(a^2+x^2) + C$$

$$\text{三十一} \quad u = \frac{(2x^2-1)(1+x^2)^{\frac{5}{2}}}{3x^3} + C$$

$$\text{三十二} \quad u = -\frac{1+7x^2}{3x^3} (1-2x^2)^{\frac{5}{2}} + C$$

$$\text{三十三} \quad u = -(2x + \frac{1}{x})(1+x^2)^{-\frac{1}{2}} + C$$

$$\text{三十四} \quad u = \log \left\{ x + (a^2+x^2)^{\frac{1}{2}} \right\} + C$$

$$\text{三十五} \quad u = -\frac{2\sqrt{2ax-x^2}}{x} + C$$

$$\text{三十六} \quad u = \frac{1}{a} \frac{\sqrt{x}}{\sqrt{2a-x}} + C$$

$$\text{三十七} \quad u = -\frac{1}{ae} \frac{\sqrt{a^2+x^2}}{x} + C$$

$$\text{三十八} \quad u = -\frac{2a+3x^3}{2a^2x(a+x^3)^{\frac{5}{2}}} + C$$

$$\text{廿四} \quad u = \frac{1}{2} \left\{ \log \frac{(1+x)^3}{\sqrt{1+x^2}} - \tan^{-1} x \right\} + C$$

$$\text{廿五} \quad u = \frac{1}{1+x} + \log \frac{\sqrt{x^2+x^2+x}}{(1+x)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{12x+1}{\sqrt{3}} + C$$

式卷三之卷

$$\text{廿九} \quad u = \frac{1}{3}a^3x^3 + \frac{1}{3}abx^6 + \frac{1}{9}b^3x^9 + C$$

$$\text{三十} \quad u = \frac{1}{4}a^3x^7 + \frac{3}{8}a^2b^2x^6 + \frac{3}{8}ab^2x^5 + \frac{1}{10}b^3x^{10} + C$$

$$\text{三十一} \quad u = \frac{1}{5}a^3x^5 + \frac{3}{8}a^2bx^8 + \frac{3}{11}ab^2x^{11} + \frac{1}{14}b^3x^{14} + C$$

$$\text{三十二} \quad u = \frac{1}{8}a^3x^6 + \frac{3}{12}a^2x^{11} + \frac{3}{14}ab^2x^{14} + \frac{1}{18}b^3x^{18} + C$$

$$\text{三十三} \quad u = 2ax - ax^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4 + C$$

$$\text{三十四} \quad u = \frac{(a+bx^2)^{\frac{7}{2}}}{7b^2} - \frac{a(a+bx^2)}{5ax^2} + C$$

$$\text{三十五} \quad u = \frac{1}{b^2} \left\{ \frac{(a+bx^2)^{\frac{7}{2}}}{7} - \frac{2a(a+bx^2)^{\frac{5}{2}}}{5} + \frac{a^2(a+bx^2)^{\frac{3}{2}}}{3} \right\} + C$$

三十一 $X_2 = \frac{2a}{2}x, -\frac{x}{2}\sqrt{2ax-x^2}$

三十二 $X_3 = \frac{5a}{3}X_2 - \frac{x^2}{3}\sqrt{2ax-x^2}$

三十三 $X_4 = \frac{7a}{4}X_3 - \frac{x^3}{4}\sqrt{2ax-x^2}$

三十四 $\int x^{-2}(2-x^2)^{-\frac{3}{2}} dx = \frac{x^{-1}(2-x^2)^{-\frac{1}{2}}}{2} + \int (2-x^2)^{-\frac{3}{2}} dx$

三十五 $\int \frac{dx}{\sqrt{a^2+x^2}} = \log(x + \sqrt{a^2+x^2})$

三十六 $\int dx \sqrt{x^2-a^2} = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \log(x + \sqrt{x^2-a^2})$

三十七 $\int (2-x^2)^{-\frac{3}{2}} dx = \frac{x}{2\sqrt{2-x^2}}$

三十八 $\int x(1+x^2)^{\frac{1}{3}} dx = -\frac{x^2}{2}(1+x^2)^{-\frac{2}{3}} + 2 \int \frac{dx}{(1+x^2)^{\frac{2}{3}}}$

或此 $x(1+x^2)^{\frac{1}{3}} dx$ 詳 = 級數 作
= 每項乘 $\frac{1}{2}$ 乘 $\frac{1}{2}$

三十九 $\int x^2(a+bx^2)^{-\frac{1}{2}} dx = (\frac{x^2}{3b} - \frac{2a}{3b^2})\sqrt{a+bx^2} + C$

四十 $\int x^4(a+bx^2)^{-\frac{1}{2}} dx = (\frac{x^4}{5b} - \frac{4ax^2}{15b^2} + \frac{8a^2}{15b^3})\sqrt{a+bx^2} + C$

十一 $u = \frac{1}{60}(1+x^2)^5(5x^2-1)$

十二 $u = \frac{x^2+2}{(1+x^2)^2}$

十三 $u = \frac{x^2}{2}(\log x - \frac{1}{2}) + C$

十四 $u = -\frac{x^2+2}{3(1+x^2)^2} + C$

十五 $u = \frac{x^{n+1} \log x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$

十六 $u = -\frac{x^2}{3}(a-x^2)^{\frac{3}{2}} - \frac{2}{15}(a-x^2)^{\frac{5}{2}} + C$

十七 $u = -\frac{(1-x^2)^{\frac{1}{2}}}{x} - \sin^{-1} x$

十八 $u = \frac{x}{2}(a^2-x^2)^{\frac{1}{2}} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

十九 $X_4 = \frac{3a^2}{4}X_2 - \frac{x^2}{4}\sqrt{a^2-x^2}$

二十 $X_6 = \frac{5a^4}{6}X_4 - \frac{x^4}{6}\sqrt{a^2-x^2}$

二十一 $X_8 = \frac{7a^6}{8}X_6 - \frac{x^6}{8}\sqrt{a^2-x^2}$

式卷四之卷

一 $u = \frac{1}{4} \log \frac{(x+2)^2}{x(x+4)} + C$

二 $u = \log \frac{x}{x+1} - \frac{2}{x+1} + C$

三 $u = \log \frac{2x+1}{x+1} + C$

四 $u = \frac{2}{25} \log \frac{x-2}{x+3} - \frac{2}{5} \frac{1}{x+3} + C$

五 $u = \frac{1}{3} \log \frac{(x-1)^3}{\sqrt{x^2+2x}} + C$

六 $u = \log \frac{(x^2+1)^{\frac{1}{2}}(x+2)^{\frac{2}{3}}}{\sqrt{x+1}} + \frac{3}{10} \tan^{-1} x$

七 $u = \frac{1}{5} \log \left(\frac{(x+1)\sqrt{x^2-x+1}}{(x+1)\sqrt{x^2+x+1}} \right) - \frac{1}{2\sqrt{3}} \left(\tan^{-1} \frac{2x-1}{\sqrt{3}} + \tan^{-1} \frac{x+2}{\sqrt{3}} \right) + C$

八 $u = \frac{4x^2+3}{4(x^2+1)} + \log \sqrt{x^2+1} + C$

一 $\int \frac{2x}{x^2\sqrt{a+bx^2}} = \left(-\frac{1}{3ax} + \frac{2b}{3a^2x} \right) \sqrt{a+bx^2} + C$

二 $\int x^{-6}(a+bx^2)^{-\frac{1}{2}} dx = \left(-\frac{1}{5ax^5} + \frac{2b}{15a^2x} - \frac{4b^2}{15a^3x} \right) \sqrt{a+bx^2}$

三 $\int (a-x^2)^{\frac{1}{2}} dx = \frac{x(a-x^2)^{\frac{1}{2}}}{2} + \frac{a}{2} \log(x+(a-x^2)^{\frac{1}{2}}) + C$

四 $\int (a+x^2)^{\frac{3}{2}} dx = \frac{x(a+x^2)^{\frac{3}{2}}}{4} + \frac{3ax(a+x^2)^{\frac{1}{2}}}{8} + \frac{3a^2}{8} \log \left\{ x+(a+x^2)^{\frac{1}{2}} \right\} + C$

五 $\int (a+bx^2)^{-\frac{3}{2}} dx = \frac{x}{a} (a+bx^2)^{-\frac{1}{2}} + C$

六 $\int (a+x^2)^{-2} dx = \frac{x}{2a(a+x^2)} + \frac{1}{2a^2} \tan^{-1} \frac{x}{a} + C$

六 $v = \log\left(\frac{x}{1-x}\right) + C$

七 $v = \log\left(\frac{x}{\sqrt{1-x^2}}\right) + C$

八 $v = \frac{1}{k-a} \log\left(\frac{x+k}{x+a}\right) + C$

九 $v = \frac{a^2}{(b-a)(c-a)} \log(x+a) + \frac{b}{(a-b)(c-b)} \log(x+b) + \frac{c^2}{(a-c)(b-c)} \log(x+c) + C$

十 $v = \frac{(x-a)\sqrt{2ax-x^2}}{2} + \frac{a^2}{2} \arctan\frac{x}{a} + C$

十一 $v = \frac{x\sqrt{x}}{a\sqrt{2a-x}} + \frac{1}{a}\sqrt{2ax-x^2} - \sqrt{2a} \arctan\frac{x}{a} + C$

十二 $v = 429 l \dots$

十三 $v = \frac{1}{4}x^4 + \log(\sqrt{x^2+1}) + C$

十四 $v = \frac{1}{2\sqrt{2}} \log\sqrt{\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}} + \frac{1}{2\sqrt{2}} \arctan\left(\frac{x\sqrt{2}}{1-x}\right) + C$

十五 $v = \frac{x^4}{4} - x + \frac{1}{3} \log\left(\frac{x+1}{\sqrt{x^2-1}}\right) + \frac{1}{\sqrt{3}} \arctan\left(\frac{2x-1}{3}\right) + C$

十六 $v = \frac{2}{3}(a+x)^{\frac{3}{2}} - \frac{2a}{3}(a+x)^{\frac{5}{2}} + C$

十七 $v = \frac{2}{3a} \left((x+a)^{\frac{3}{2}} + \sqrt{\frac{2}{a}} \right) + C$

十八 $v = \frac{2}{\sqrt{a}} \log\frac{\sqrt{x}}{\sqrt{ax+a}} + C$

十九 $v = -\frac{x(b-x)^{\frac{m}{n}+2}}{m+n} - \frac{x(a-b)(b-x)^{\frac{m}{n}+1}}{m+n}$

二十 $v = \frac{x}{4} (x^2+4a)^{\frac{5}{2}} + C$

二十一 $v = \frac{2\sqrt{a+bx^3}}{5} - \frac{2\sqrt{a}}{3} \log\frac{\sqrt{b+ax^2}+\sqrt{ax^3}}{\sqrt{b}} + C$

二十二 $v = \sqrt{x^2-a^2} - a \arctan\frac{\sqrt{x^2-a^2}}{a} + C$

三十五 $u = \log(x+1) + \frac{4}{x+2} + C$

三十六 $u = \log \frac{\sqrt{x^2+1}}{x+1} + \tan^{-1} x + C$

三十七 $u = \frac{1}{8} \frac{1}{x+1} + \frac{1}{9} \log \frac{x+1}{\sqrt{x^2-2x+3}} + \frac{1}{13\sqrt{2}} \tan^{-1} \frac{x-1}{2} + C$

三十八 $u = \frac{1}{\sqrt{2}} \log \frac{\sqrt{2x+\sqrt{1+x^2}}}{1-x^2} + C$

三十九 $u = \log \{2\sqrt{x^2+1} + 2x+1\} + C$

四十 $u = -\frac{1}{x} + \log \frac{\sqrt{x^2+1}}{x} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x+1}{\sqrt{3}} + C$

四十一 $u = \frac{1}{3} \left\{ 5\sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right) - \frac{5}{x} \right\} + C$

四十二 $u = \frac{3-2x-5x^2}{4x^2(1+x)} + \log \sqrt{\frac{(x+1)(x-1)}{(1+x^2)x^2}} - \frac{1}{4} \tan^{-1} x + C$

四十三 $u = \frac{x^2}{9} - \frac{2x}{9} + \frac{2}{9} \sqrt{\frac{2}{3}} \tan^{-1} x \sqrt{\frac{3}{2}} + C$

四十四 $u = \frac{1}{4} a^x \pi$

四十五 $u = \frac{x^2}{3} \left\{ (\log x)^2 - \frac{2}{3} \log x + \frac{2}{9} \right\} + C$

四十六 $u = -\frac{5+2x}{4(x+1)^2} + \frac{1}{16} \log \left(\frac{x-3}{x+1} \right) + C$

四十七 $u = \frac{3x+2}{x^2+3x+2} + \log \left(\frac{x+1}{x+2} \right)^3 + C$

四十八 $u = \frac{a^x}{2} \frac{x}{2}$

四十九 $u = \frac{2}{5} x^{\frac{5}{2}} - \frac{2}{3} x^{\frac{3}{2}} + x^{\frac{1}{2}} - 2 \tan^{-1} x^{\frac{1}{2}} + C$

五十 $u = (x+\sqrt{2})^2 + C$

五十一 $u = \frac{2ab^2+ek^4}{6} + C$

五十二 $u = \frac{1}{22} \left\{ \log \left(\frac{(x+2)^3}{(x+1)\sqrt{x^2+3}} \right) + 2\sqrt{3} \tan^{-1} \frac{x}{\sqrt{3}} \right\} + C$

壬
$$u = -\frac{x+2}{3(1+x^2)} - \frac{2}{2\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

壬
$$u = \frac{x^3}{3} - \log \sqrt[3]{x^3+1}$$

壬
$$u = \log \sqrt[4]{\frac{7+x}{7-x}} - \frac{1}{2} \tan^{-1} x$$

癸
$$u = \frac{1}{4a} \log \frac{x-a}{x+a} + \frac{1}{2a} \tan^{-1} \frac{x}{a}$$

癸
$$u = \log \sqrt{\frac{1+x^2}{1-x^2}}$$

癸
$$u = \tan^{-1}(x^3)$$

癸
$$u = \tan^{-1}(2x^2+1)$$

癸
$$u = \log \frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1} + \frac{1}{2\sqrt{2}} \tan^{-1} \frac{x\sqrt{2}}{1-x^2}$$

癸
$$u = \frac{x^2}{2} + \frac{1}{3} \log \frac{\sqrt{x^2-x+1}}{(x+1)} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{x\sqrt{3}}{2-x}$$

甲
$$u = \frac{1}{2ae} \tan^{-1} \frac{x^e}{ae} + C$$

甲
$$u = \frac{x^4}{2} + \frac{x^e}{2} + \frac{1}{8} \log \sqrt{1+x^e} + C$$

甲
$$u = \frac{1}{(1+x^2)} \left(x^2 + \frac{3x}{2}\right) - \frac{3}{2} \tan^{-1} x + C$$

甲
$$u = -\frac{x^2}{4} \frac{3x^2+2}{(1+x^2)^2} + \log \sqrt{1+x^2} + C$$

甲
$$u = -\frac{1}{x^5} + \frac{1}{3x^3} - \frac{1}{x} - \tan^{-1} x + C$$

甲
$$u = -\left(\frac{1}{3x^2} - \frac{5}{3x} - \frac{5x}{2}\right) \frac{1}{1+x^2} + \frac{5}{2} \tan^{-1} x + C$$

甲
$$u = \frac{2}{\sqrt{3}} \tan^{-1} \frac{1+2x}{\sqrt{3}} + C$$

甲
$$u = \frac{1}{\sqrt{3}} \log \left\{ \frac{2x+1-\sqrt{5}}{2x+1+\sqrt{5}} \right\} + C$$

甲
$$u = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2(b+ax)}{\sqrt{4ac-b^2}} + C$$

七二
$$u = \frac{2}{\sqrt{a}} \operatorname{arctg} \sqrt{\frac{bx-a}{a}} = \frac{2}{\sqrt{a}} \operatorname{arctg} \sqrt{\frac{bx-a}{a}}$$

七三
$$u = -\frac{\sqrt{a+bx}}{a} + \frac{b}{2\sqrt{a}} \operatorname{arctg} \left\{ \frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}} \right\}$$

七四
$$u = -\frac{\sqrt{4+3x}}{2x} - \frac{3}{10} \operatorname{arctg} \frac{\sqrt{4+3x}-2}{\sqrt{4+3x}+3}$$

七五
$$u = \frac{2}{\sqrt{1+x}} \left\{ x^2 - 4x - 8 \right\}$$

七六
$$u = \frac{2}{a(a+bx)^2} + \frac{1}{a\sqrt{a}} \operatorname{arctg} \left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}} \right)$$

七七
$$u = (x^2 - 6x - 9) \frac{3}{4(1+x)^{5/2}}$$

七八
$$u = \frac{2+3x}{3} \frac{2}{(1+2x)^2} + \operatorname{arctg} \left(\frac{\sqrt{1+2x}-1}{\sqrt{1+2x}+1} \right)$$

七九
$$u = -\left(x^3 + 4x^2 + \frac{31x}{5} + \frac{124}{35} \right) \frac{1}{(2+x)^{7/2}}$$

八〇
$$u = (2+3x)^{-2/3} \left\{ \frac{7-4x+5x^2}{40} \right\}$$

八一
$$u = \frac{1}{5} \operatorname{arctg} \frac{(2-1)\sqrt{x^2-1}}{(x+1)\sqrt{x+1}} - \frac{1}{2\sqrt{5}} \operatorname{arctg} \frac{x\sqrt{5}}{1-x^2}$$

八二
$$u = \frac{1}{9} \operatorname{arctg} 2 - \frac{3}{4} + \frac{\pi}{5\sqrt{5}}$$

八三
$$u = \frac{1}{2(1+x^2)} = \operatorname{arctg}^4 \sqrt{\frac{x^2}{7+x^2}}$$

八四
$$u = -\frac{1}{30x^3} + \frac{1}{3x^2} \operatorname{arctg} \left(\frac{a+bx}{x^2} \right)$$

八五
$$u = \frac{1}{3(1+x^2)} - \operatorname{arctg}^3 \sqrt{\frac{1+x}{x^2}}$$

八六
$$u = \left(\frac{(a+bx)^2}{7} - \frac{2a(a+bx)}{5} + \frac{a^2}{3} \right) \frac{2(a+bx)^{3/2}}{b^2}$$

八七
$$u = \left(\frac{(a+bx)^3}{7} - \frac{3}{5} a(a+bx)^2 + a^2(a+bx) - a^3 \right)$$

八八
$$u = \operatorname{arctg}^3 \sqrt{\frac{x^2}{1+x^2}}$$

八九
$$u = \frac{1}{\sqrt{a}} \operatorname{arctg} \frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}$$

$$\text{八〇} \quad u = (14x^3 - 12x^2 + 27x - 81) \frac{2\sqrt{1+x}}{4 \cdot 5 \cdot 7}$$

$$\text{八一} \quad u = \left(x^4 - \frac{4x^2}{5} + \frac{2}{15}\right) \frac{(1+x^2)^{\frac{3}{2}}}{7}$$

$$\text{八二} \quad u = -\left(\frac{1}{x^2} + \frac{2}{3x}\right) \frac{(1+x^2)^{\frac{5}{2}}}{5}$$

$$\text{八三} \quad u = \frac{5x^2 - 2}{35} (1+x^2)^{\frac{5}{2}}$$

$$\text{八四} \quad u = \frac{7x^2 - 2}{15} (1+x^2)^{\frac{7}{2}}$$

$$\text{八五} \quad u = -\left\{\frac{x^4}{5} + \frac{2x^2}{15} + \frac{8}{15}\right\} \sqrt{1-x^2}$$

$$\text{八六} \quad u = -\left\{\frac{x^5}{6} + \frac{5x^3}{24} + \frac{5x}{18}\right\} \sqrt{1-x^2} + \frac{5}{18} \sin^{-1} x$$

$$\text{八七} \quad u = -\left\{\frac{1}{5x^2} - \frac{2}{15x} + \frac{8}{15x}\right\} \sqrt{1+x^2}$$

$$\text{八八} \quad u = \left\{\frac{1}{3a(a+x^2)} + \frac{2}{3a^2}\right\} \frac{x}{\sqrt{a+x^2}}$$

$$\text{八九} \quad u = \frac{3}{2} \left\{ \frac{\sin^{-1} x \sqrt{2}}{\sqrt{2}} + x\sqrt{1-2x} \right\} + \frac{1}{4} x(1-2x)^{\frac{3}{2}}$$

$$\text{九〇} \quad u = \frac{x^3 + 3x}{2} \frac{1}{\sqrt{1+x^2}} - \frac{3}{2} \log(x + \sqrt{1+x^2})$$

$$\text{九一} \quad u = \frac{x^2 + 3x}{2} \frac{1}{\sqrt{1+x^2}} - \frac{3}{2} \log(x + \sqrt{1+x^2})$$

$$\text{九二} \quad u = -\left(x^2 + \frac{2}{3}\right) \frac{1}{(1+x^2)^{\frac{5}{2}}}$$

$$\text{九三} \quad u = \left(x^4 + 4x^2 + \frac{4}{3}\right) \frac{1}{(1+x^2)^{\frac{7}{2}}}$$

$$\text{九四} \quad u = \left(\frac{8x^3}{105} + \frac{2x^5}{15} + \frac{x^3}{3}\right) \frac{1}{(1+x^2)^{\frac{7}{2}}}$$

$$\text{九五} \quad u = -\tan^{-1}\left(\frac{\sqrt{1-2x}}{x}\right)$$

$$\text{九六} \quad u = 2 \tan^{-1} \sqrt{1+x}$$

$$\text{九七} \quad u = \log(2x + 1 + 2\sqrt{1+x+x^2})$$

$$\text{一七} \quad u = \log\left(\frac{x+2-2\sqrt{1-x^2}}{1+x}\right)$$

$$\text{一八} \quad u = \operatorname{Cot}^{-1}\left(\frac{1-x}{\sqrt{x}}\right)$$

$$\text{一九} \quad u = \frac{1}{2} \sin^{-1}\left(\frac{x^2}{a^2}\right)$$

$$\text{二〇} \quad u = \sin^{-1}\sqrt{\frac{x^2-a^2}{b^2-a^2}}$$

$$\text{二一} \quad u = \frac{x}{a\sqrt{a^2-x^2}}$$

$$\text{二二} \quad u = \sin^{-1}x - \sqrt{1-x^2}$$

$$\text{二三} \quad u = \sin^{-1}x + \log\left(\frac{x}{1+\sqrt{1-x^2}}\right)$$

$$\text{二四} \quad u = \frac{1}{2} \sin^{-1}x - \frac{1}{2}(2+x)\sqrt{1-x^2}$$

$$\text{二五} \quad u = \frac{2}{3} \tan^{-1}\sqrt{\frac{x^2}{a^2-x^2}}$$

$$\text{九八} \quad u = \sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)$$

$$\text{九九} \quad u = \frac{1}{\sqrt{2}} \log\left(\frac{x-2+3\sqrt{2}\sqrt{1-x}}{1+x}\right)$$

$$\text{百} \quad u = \log\left(\frac{2+x-2\sqrt{1+x^2}}{x}\right)$$

$$\text{一〇一} \quad u = \sin^{-1}\left(\frac{x-2}{3\sqrt{5}}\right)$$

$$\text{一〇二} \quad u = \frac{1}{2} \log\left(\frac{1-x\sqrt{2}\sqrt{1-x^2}}{1+x}\right)$$

$$\text{一〇三} \quad u = \frac{1}{\sqrt{2}} \operatorname{Cot}^{-1}\sqrt{\frac{1-x^2}{1+x}}$$

$$\text{一〇四} \quad u = \frac{1}{\sqrt{2}} \log\left(\frac{\sqrt{1+x}+x\sqrt{2}}{\sqrt{1-x^2}}\right)$$

$$\text{一〇五} \quad u = \tan^{-1}\left(\frac{1+3x}{2\sqrt{1-x^2}}\right)$$

$$\text{一〇六} \quad u = \log\sqrt{x^2+\sqrt{a^2+x^2}}$$

$$\text{二五} \quad u = \frac{1}{2} \tan^{-1} \sqrt{\frac{1-x^2}{1+x^2}} - \frac{1}{4} (2x+x^2) \sqrt{1-x^2}$$

$$\text{二六} \quad u = \frac{1}{\sqrt{a-2}} \cos^{-1} \left(\frac{x\sqrt{a-2}}{x^2-1} \right)$$

$$\text{二七} \quad u = \frac{1}{\sqrt{2}} \sin^{-1} \frac{x\sqrt{2}}{1+x^2}$$

$$\text{二八} \quad u =$$

$$\text{二九} \quad u =$$

$$\text{三〇} \quad u = \frac{1}{2} (\log x)^2$$

$$\text{三一} \quad u =$$

$$\text{三二} \quad u =$$

$$\text{三三} \quad u = a^x \left\{ \frac{x^5}{A} - \frac{2x^2}{A^2} + \frac{6x}{A^3} - \frac{6}{A^4} \right\}$$

$$\text{二六} \quad u = -\frac{x+a}{a^2 \sqrt{2ax+x^2}}$$

$$\text{二七} \quad u = -\left\{ \frac{1}{2(2ax+x^2)} - \frac{2}{3a^2} \right\} \frac{x+a}{a^2 \sqrt{2ax+x^2}}$$

$$\text{二八} \quad u = \frac{2(2x+1)}{3\sqrt{1+x+x^2}}$$

$$\text{二九} \quad u = \left\{ \frac{1}{1+x+x^2} + \frac{8}{3} \right\} \frac{2(2x+1)}{3\sqrt{1+x+x^2}}$$

$$\text{三〇} \quad u = \frac{\sqrt{x}}{1+x} + \tan^{-1}(\sqrt{x})$$

$$\text{三一} \quad u = \left(\frac{x}{3} - 1 \right) 2\sqrt{x} + 2 \tan^{-1} \sqrt{x}$$

$$\text{三二} \quad u = 2\sqrt{x} + \frac{1}{\sqrt{2}} \left\{ \log \left(\frac{x+1+\sqrt{2x}}{\sqrt{1+x^2}} \right) - \tan^{-1} \frac{\sqrt{2x}}{1-x} \right\}$$

$$\text{三三} \quad u = \frac{\sqrt{2ax-x^2}}{3a^2 x^2} (a+x)$$

$$\text{三四} \quad u = \sqrt{a+bx+cx^2} \left\{ \frac{2cx-2b}{4c^2} \right\} + \frac{2ac-2b^2}{8c^2 \sqrt{c}} \log \left(\frac{2cx+2b}{2c} \right)$$

一四一

$$u = l^x \sqrt{\frac{1+x}{1-x}}$$

一四二

$$u = \log \frac{l^x}{l^x - 1} - \frac{1}{l^x - 1} \left\{ 1 + \frac{x}{2(l^x - 1)} \right\}$$

一四三

$$u = -\frac{1}{3}(\sin \theta)^2 \cos \theta - \frac{2}{3} \cos \theta$$

一四四

$$u = \frac{1}{3}(\cos \theta)^2 \sin \theta + \frac{2}{3} \sin \theta$$

一四五

$$u = -\cos \theta \left\{ \frac{(\sin \theta)^4}{5} + \frac{4(\sin \theta)^2}{15} + \frac{8}{15} \right\}$$

一四六

$$u =$$

一四七

$$u =$$

一四八

$$u = \frac{(\sin \theta \cos \theta)^5}{6} + \frac{1}{8} \sin^3 \theta \cos \theta - \frac{1}{10} \sin \theta \cos \theta$$

一四九

$$u = \left\{ \frac{(\cos \theta)^2}{9} + \frac{2}{63} \right\} (\sin \theta)^7$$

一五〇

$$u =$$

一五一

$$u =$$

一五二

$$u = 2x^{\frac{1}{2}} \left\{ x^{\frac{3}{2}} - 2x + 6\sqrt{x} - 6 \right\}$$

一五三

$$u = a^x \left\{ \frac{1}{3x^3} + \frac{1}{2x^2} + \frac{A^2}{2.3x} \right\} + \frac{A^3}{2.3} \int \frac{a^x}{x}$$

一五四

$$u = \frac{a^x}{Ax} \left\{ 1 + \frac{1}{2xA} + \frac{5}{(2xA)^2} + \frac{3x}{(2xA)^3} + \dots \right\}$$

一五五

$$u = \frac{x}{x+1} \log x - \log(1+x)$$

一五六

$$u = \log x \sqrt{1+x}$$

一五七

$$u = \log \left(\frac{l^x}{l^x + 1} \right)$$

一五八

$$u = l^x \left(\frac{x-1}{x+1} \right)$$

$$\text{一五二} \quad u = \frac{-1}{4(\tan \theta)^4} + \frac{1}{2(\tan \theta)^2} + \log(\sin \theta)$$

$$\text{一五三} \quad u =$$

$$\text{一五四} \quad u =$$

$$\text{一五五} \quad u = \frac{\sin^{-1} x}{\sqrt{1-x^2}} + \log \sqrt{\frac{1-x}{1+x}}$$

$$\text{一五六} \quad u =$$

$$\text{一五七} \quad u = \frac{x^3 + ax^3}{3} \tan^{-1} \sqrt{\frac{x}{a}} - \frac{\sqrt{ax} \left(\frac{x^2}{3} - \frac{ax}{3} + a^2 \right)}{3}$$

$$\text{一五八} \quad u =$$

$$\text{一五九} \quad u =$$

$$\text{一六〇} \quad u = \frac{e^{ax} \sin x (a \sin x + \cos x)}{a^2 + 1} + \frac{e^{-ax}}{a(a^2 + 1)}$$

$$\text{一六一} \quad u = -\frac{1}{2} \frac{\cot \theta}{\sin \theta} + \frac{1}{2} \log(\tan \frac{\theta}{2})$$

$$\text{一六二} \quad u =$$

$$\text{一六三} \quad u =$$

$$\text{一六四} \quad u = \frac{1}{(\cot \theta)} \left\{ (\sin \theta)^2 - \frac{e}{3} \right\}$$

$$\text{一六五} \quad u =$$

$$\text{一六六} \quad u =$$

$$\text{一六七} \quad u = \left\{ \frac{1}{2(\cot \theta)^2} - \frac{3}{2} \right\} \frac{1}{\sin \theta} + \frac{3}{2} \log \tan \left(\frac{x}{4} + \frac{\theta}{8} \right)$$

$$\text{一六八} \quad u = -\frac{1}{3 \cos \theta (\sin \theta)^2} - \frac{8}{3} \cot 2\theta$$

$$\text{一六九} \quad u = \frac{1}{3} \tan^3 \theta - \tan \theta + \theta$$

式答五之卷

一 $u = \frac{a}{24}x^4 + \frac{cx^2}{2} + c'x + c''$

二 $u =$

三 $u = 6x^4 + \frac{cx^3}{6} + \frac{c'x^2}{2} + c'x + c''$

四 $u = \frac{80x^{\frac{7}{2}}}{105} + \frac{cx^2}{2} + c'x + c''$

五 $u = \frac{c_1 x^4}{8} + \frac{c_2 x^3}{6} + \frac{c'x^2}{2} + c'x + c''$

六 $2y^{\frac{1}{2}} = \tan^{-1}x + c$

七 $x^2y - 2 = 20y$

八 $(1+x)(1+y) = c$

一 $v = \frac{1}{\sqrt{ab}} \tan^{-1} \left(\sqrt{\frac{b}{a}} \tan \theta \right)$

二 $u = \frac{1}{1-c^2} \frac{\sin \theta}{\sqrt{1-c^2 \cos^2 \theta}}$

三 $u =$

四 $u =$

五

六 $u = \left\{ 1 + A^2 + \left(\frac{A^2}{2}\right)^2 + \left(\frac{A^2}{2.3}\right)^2 + \dots \right\} x$

式答七之卷

二

$$三 \quad z = \frac{r}{2\sqrt{x}} \sqrt{(x^2+y^2)} + \frac{1}{2} \log\left(\frac{r+\sqrt{(x^2+y^2)}}{r}\right)$$

三 $z = r\sqrt{2}$

四 $s = \frac{\pi}{n+1} xy$

五 $s = 3.1416904 \dots$

六 $s = a, b, \pi$

七 $r = xy - a, b, \log\left(\frac{ay+bx}{ab}\right)$

八

九 $xy^2 = a$

+ $\frac{x}{y} = \frac{y^2}{2} + y + c$

± $y^2 - x^2 = 2axy + c$

± $x + \frac{ly}{2a} = \frac{(4ay+bx)^{\frac{3}{2}}}{12a^2} + c$

± $y^2 - 2x^2 = ly^{\frac{2}{3}}$

± $y^2 x = (2y+x)a^2$

± $\log x = -\frac{1}{n} + c = -\frac{x}{y} + c$

$$\text{八} \quad \beta = M$$

$$\text{七} \quad \beta = \frac{x^2}{2} + \frac{x^3}{3}$$

$$\text{六} \quad \beta = \frac{a^2}{3}$$

$$\text{五} \quad \beta = \frac{17P}{2 \cdot 3} + \frac{77P^2}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{25577P^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \dots$$

九

十

$$\text{十一} \quad Z = y + \frac{7P}{2 \cdot 3} + \frac{77P^2}{2 \cdot 2 \cdot 4 \cdot 5} + \frac{7 \cdot 2 \cdot 5 \cdot 7 P^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \dots \frac{\beta^2}{\pi} = P$$

$$\text{十二} \quad \beta = m$$

$$\text{十三} \quad \beta = y + ax$$

$$\text{十四} \quad \beta = a^2 + b^2 + ax + \log ab$$

$$\text{十五} \quad \beta =$$

$$\text{十六} \quad \beta =$$

$$\text{十七} \quad \beta = \log ab + ab + \frac{n}{2} ab$$

$$九 \quad V = \frac{1}{2} \pi r^2 x$$

$$十 \quad V = \frac{5}{2} r^3 x^2$$

式卷十之卷

$$一 \quad A = a^2 \left(\tan \frac{\theta}{2} + \frac{1}{3} \tan^3 \frac{\theta}{2} \right)$$

$$二 \quad A = \frac{3}{2} a^2$$

$$三 \quad A = \frac{3\alpha\beta}{8} \frac{\pi}{4} \quad \therefore 4A = \frac{3\pi}{8} \frac{(a^2 - b^2)^2}{ab}$$

$$四 \quad V = \frac{4}{3} \pi a^3$$

$$五 \quad V = \pi b^2 \left\{ \frac{\pi a}{2} + \frac{2b}{3} \right\}$$

式卷八之卷

$$一 \quad \beta = -\frac{a^2}{at}$$

$$二 \quad \beta = 2\pi b h$$

$$三 \quad \beta = 4\pi R^2$$

$$四 \quad \beta = \frac{2\pi}{3\sqrt{3}} \left((a^2 + b^2)^{\frac{3}{2}} + b^3 \right)$$

$$五 \quad \beta = \frac{6\pi}{5} \pi r^2$$

$$六 \quad V = (\pi r^2 h)^{\frac{1}{3}}$$

$$七 \quad V = \frac{2}{3} \pi b^2 x 2a$$

$$八 \quad V = \frac{\pi}{n+2} \pi r^2 x$$

$$\text{面積} = \left(\frac{\pi}{2} - 1\right) 4r^2 = (2\pi - 4)r^2$$

$$\text{十 四 體積} = 4r^2 \frac{\pi}{4} \sqrt{2} = r^2 \pi \sqrt{2}$$

$$\text{內面積} = \left(\frac{\pi}{4} - \frac{1}{3}\right) 8r^3$$

$$\text{十 五 面積} = \sqrt[3]{\left(1 - \frac{\pi}{4}\right)} V$$

$$\text{• 傍面積} = 2r^2 k$$

$$\text{十 六 截面積} = \frac{4}{3} r^2 k$$

$$\text{截面積} = (\sqrt{4r^2 + k^2}) r$$

$$\text{左右實積} = \left(\frac{4b^2}{3c} + c\right) b r \frac{1}{4}$$

$$\text{十 七 左右面積} = \left(\frac{2b^2}{c} + \frac{c}{2}\right) k$$

$$\text{六 } V = \pi^2 a^3$$

$$\text{七 } V = \frac{8}{15} \pi a^2 h$$

$$\text{八 } s =$$

$$\text{九 } s = 4ab\pi^2$$

$$\text{十 } z = 6a$$

$$\text{十一 } A = a\sqrt{y^2 - a^2}$$

$$\text{十二 } A = ab + (a^2 - b^2) \tan^{-1}\left(\frac{a}{b}\right)$$

$$\text{十三 } s = 4a^2 = s'$$

$$\text{十三 } V = \frac{16}{3} a^2$$

本面積

本家積

内面積

面積 = $7(\sqrt{8}-2)r^2$

家積 = $8(\sqrt{2}-\frac{2}{3})r^2$

内面積 = $\sqrt{8} \cdot 4r^2$

傍面積 = $(\frac{4r^2}{2} + 2r^2) \cdot 4$

等面積 = $(\frac{1}{2}(2r+2)-2) \cdot 2r$

面積

内面積

本家積

本面積

家積

本面積

白面積

白面積

黑面積

黑面積

白面積 = $3\sqrt{4r^2+r^2} \div (2r^2+\sqrt{4r^2+r^2})$

白面積 = $(\sqrt{4r^2+r^2}) \div 2 - \frac{r^2}{4}$

黑面積 = $(\frac{1}{2}(16r^2+r^2)-r^2) \cdot \frac{r}{4r^2}$

黑面積 = $(\sqrt{4r^2+r^2}-r) \cdot r$

家積

内面積











